Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Limits of Computational Learning

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/learning/

Assignment 9 Deadline: Thu 19.7.2012, 10am

The first exercise of this sheet concerns *optimal identification*. We make the following definition. Given a learner h and a learnee g, we let $\mathrm{conv}(h, g)$ be the least t such that, for all $t' > t$, $h(g[t']) = h(g[t])$ (the point where h on g has converged). Note that conv is not computable. A learner h is said to be *optimal* iff, for all learners h' learning every function **GEx**-learned by h, if there is $g \in \mathbf{GEx}(h)$ with $\mathrm{conv}(h', g) < \mathrm{conv}(h, g)$, then there is a $g' \in \mathbf{GEx}(h)$ with $\mathrm{conv}(h, g') < \mathrm{conv}(h', g')$; in other words, if h' is strictly better than h on some target function, then h' is strictly worse than h on some other target function.

Exercise 1 *(8pts, 4pts for each direction) Let $h \in \mathcal{P}$ show that h is optimal iff*

(a) h is consistent (on what it learns);

(b) h is prudent (on what it learns); and

(c) h is strongly non-U-shaped (on what it learns).

Exercise 2 *(8pts, 4pts for each direction) Let $\mathcal{S} \subseteq \mathcal{R}$. Show that $\mathcal{S} \in$ **GFin** is equivalent to the existence of $p, d \in \mathcal{R}$ such that*

(a) $\mathcal{S} \subseteq \{\varphi_{p(i)} \mid i \in \mathbb{N}\}$; and

(b) For all i, j with $i \neq j$ we have

$$\exists x \leqslant d(i) : \varphi_{p(i)}(x) \neq \varphi_{p(j)}(x).^{1}$$

Explicitly show that **GFin** *allows for learning by enumeration.*

[1] Note that $d(i)$ thus give a bound independent of j.