This homework set has three questions, each one with increasing difficulty. You must work in pairs to determine the solutions.

• Every member of the team must be able to explain how you arrived at the answer.

• You may be asked to present your answer on the blackboard.

1. Let $G$ be an infinite graph and $A, B \subseteq V(G)$. Show that if no finite set of vertices seperate $A$ from $B$ in $G$, then $G$ contains an infinite set of disjoint $A - B$ paths.

2. Show that the $k \times k$ grid has tree-width at least $k$.

3. Show that if a graph has circumference $k \neq 0$, then its tree-width is at most $k - 1$. 