• This homework set has three questions, each one with increasing difficulty. You must work in pairs to determine the solutions.

• Every member of the team must be able to explain how you arrived at the answer.

• You may be asked to present your answer on the blackboard.

1. Let $k \geq 2$. Show that in a $k$-connected graph any $k$ vertices lie on a common cycle.

2. Let $G$ be a $k$-connected graph, and let $xy$ be an edge of $G$. Show that $G/xy$ is $k$-connected if and only if $G - \{x, y\}$ is $(k-1)$-connected.

3. Find the error in the following ‘simple proof’ of Menger’s theorem. Let $X$ be an $A - B$ separator of minimum size. Denote by $G_A$ the subgraph of $G$ induced by $X$ and all the components of $G - X$ that meet $A$, and define $G_B$ correspondingly. By the minimality of $X$, there can be no $A - X$ separator in $G_A$ with fewer than $|X|$ vertices, so $G_A$ contains $k$ disjoint $A - X$ paths by induction. Similarly, $G_B$ contains $k$ disjoint $X - B$ paths. Together, all these paths form the desired $A - B$ paths in $G$. 

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