- This homework set has *three* questions, each one with increasing difficulty. You must work in pairs to determine the solutions.
- Every member of the team must be able to explain how you arrived at the answer.
- You may be asked to present your answer on the blackboard.

1. Prove by induction that every cut $(S, \overline{S})$ in N satisfies $f(S, \overline{S}) = f(s, V)$

2. Consider a graph in which each edge has both a minimum $c_{min}(x, y)$ and maximum $c_{max}(x, y)$ capacity. A flow f is legal, if

$$c_{min}(x, y) \leq f(x, y) \leq c_{max}(x, y)$$

Note that we no longer require that $f(x, y) = -f(y, x)$ and it might be that both $f(x, y)$ and $f(y, x)$ are positive.

(a) suppose that we are given some legal flow f. Give an algorithm that either produces a larger legal flow or recognises that f is a maximal legal flow

(b) Is there an algorithm such that for a graph G with minimum and maximal capacities either finds a legal flow or recognises that there is no such flow?