1. A k-chromatic graph is called critically k−chromatic, or just critical, if χ(G−v) < k for every v ∈ V(G). Show that every k-chromatic graph has a critical k-chromatic induced subgraph, and that any such subgraph has a minimum degree at least k − 1.

2. Determine the critical 3-chromatic graphs.

3. Show that the following statements are equivalent for a graph G:
   1. χ(G) ≤ k
   2. G has an acyclic orientation without directed paths of length k.

4. **Show that the following statements are equivalent for a graph G:
   1. χ(G ≤ k
   2. G has an orientation without directed paths of length k
   3. G has an acyclic such orientation (one without directed cycles).**