- This homework set has *four* questions, each one with increasing difficulty. You must work in pairs to determine the solutions.
- Every member of the team must be able to explain how you arrived at the answer.
- You may be asked to present your answer on the blackboard.

1. Is the dual graph G^* connected for every planar (not necessarily connected) graph G? Find a counterexample or proof.
2. Let G be a planar embedding. Let A be a set of lists, one for each face of G, such that each list contains all the edges of its face in clockwise order. Show that A and a combinatorial embedding are equivalent in the sense that they define each other.
3. Prove the following statement or show a contradiction: A simple planar graph with $n \geq 2$ contains at least 2 vertices that have degree at most 5, respectively.
4. Draw an arbitrary closed curve on a paper. The curve may intersect itself; however, every self-intersection must be a point. It is also possible that more than two parts of the curve intersect at one point. Show that the resulting regions on the paper can be colored with the colors black and white such that no neighbored regions have the same color (two regions are neighbored if their intersection contains a point that is no self-intersection).