- This problemset has *six* questions.
- To get the credit for questions marked as SPOJ, you must get them accepted on `http://www.spoj.com/AOS`, but you **don't** have to send any explanation!
- For other questions, either send the solutions to `gawry1+aos@gmail.com`, or leave them in the envelope attached to the doors of my office (room 321).

1. Prove the following: a polynomial $f(x)$ of degree d over the integers modulo a prime p cannot have more than d different roots. For instance, $f(x) = x^3 + 2x + 2$ has just two roots $\{2, 3\}$ modulo 7.

(SPOJ) 2. Given q and r, compute the signature (as defined during the lecture) of a string S. The string consists of letters `a` and `b`, and we treat them as digits 0 and 1, i.e., the signature of a string `baba` is $(r^3 + r) \pmod q$.

(SPOJ) 3. We consider a different scheme for computing the signatures $\phi(S[1..n]) = (S[1]*r^{m-1}$ `xor` $S[2]*r^{m-2}$ `xor` `...` `xor` $S[n] * r^0) \bmod 2^{32}$. The string consists of letters `a` and `b`, and we again treat them as digits 0 and 1. Given r and a string S, find a different S' with exactly the same signature.

4. Compute the values of the π function for the word `ababbabaabaab`.

5. Consider a modification of the failure function π known as the strong failure function π'. It is defined as follows: for each $i = 1, 2, \ldots, |w| - 1$ we choose $\pi'[i]$ to be the longest proper border of $w[1..i]$ such that $w[\pi'[i] + 1] \neq w[i + 1]$. If there is no such border, $\pi'[i] = -1$.
   (a) Compute the values of the π' function for the word `ababbabaabaab`.
   (b) Show how to (quickly) compute the values of π' given the values of π.

6. Consider a simplification of the Boyer-Moore algorithm, where we use only the bad character rule. Show an infinite family of instances on which such modification has quadratic running time.