- This problemset has *four* questions.
- To get the credit for questions marked as SPOJ, you must get them accepted on `http://www.spoj.com/AOS`, but you **don't** have to send any explanation!
- For other questions, either send the solutions to `gawry1+aos@gmail.com`, or leave them in the envelope attached to the doors of my office (room 321).

1. During the lecture we saw a proof of the periodicity lemma. The goal of this problem is to prove a stronger version of the lemma without using the polynomials. The stronger version is: if a word w has two periods p and q, and moreover $p + q \leq |w| + \gcd(p, q)$, then $\gcd(p, q)$ is a period of w.
   (a) Show that if $p > q$ then $\gcd(p, q) = \gcd(q, p - q)$.
   (b) Assume w has periods p and q with $p > q$. Draw a picture denoting the situation.
   (c) Write down (in a formal way) the conditions that p is a period, q is a period, and $p - q$ is period. Formal means that it should begin with "p is a period of w iff for all positions i ...".
   (d) Using the above conditions, deduce that p and q being both a period of w imply that $w[i] = w[i + p - q]$ for all $i = 1, 2, \ldots, |w| - p$.
   (e) Similarly, use the conditions to deduce that p and q being both a period of w imply that $w[i] = w[i + p - q]$ for all $i = |q| + 1, |q| + 2, \ldots, |w| - (p - q)$.
   (f) Finally, notice that the two last points allow us to conclude that $w[i] = w[i + p - q]$ for all $i = 1, 2, \ldots, |w| - (p - q)$ assuming that $p + q \leq |w|$. Be very precise about using this assumption. Conclude that $p - q$ is a period of w.
   (g) Use induction to show that by iterating the above reasoning we get that $\gcd(p, q)$ is a period of w.
   (h) For extra credit: modify the above proof so that instead $p + q \leq |w|$ we only assume $p + q \leq |w| + \gcd(p, q)$.

(SPOJ) 2. Given two texts x and y, find the length of their shortest common supersequence, i.e. shortest text s such that both x and y are subsequences of s.

(SPOJ) 3. Given two texts x and y, find their shortest common supersequence, i.e. shortest text s such that both x and y are subsequences of s. In case there are many such texts, output the smallest lexicographically.

(SPOJ) 4. We say that a 2-dimensional, rectangular word w of size $n \times m$ (imagine it as a board with letter written in the squares) can be tiled with a rectangular pattern p if there are such occurrences of p in w (but not necessarily all of them) that no two of them overlap and each symbol (square) of w is covered by one of them. Given such word w, find a rectangular pattern p of smallest size (area) which the word w can be tiled with.