

- This problemset has *four* questions.
- To get the credit for questions marked as SPOJ, you must get them accepted on <http://www.spoj.com/AOS>, but you **don't** have to send any explanation!
- For other questions, either send the solutions to [gawry1+aos@gmail.com](mailto:gawry1+aos@gmail.com), or leave them in the envelope attached to the doors of my office (room 321).

1. Let  $d(i, j)$  be the edit distance between  $s[1..i]$  and  $t[1..j]$ .
    - (a) Prove that  $d(i + 1, j + 1) \geq d(i, j)$ .
    - (b) Prove that  $d(i + 1, j + 1) \leq d(i, j) + 1$ .
  2. Describe how to combine the Hirschberg's and Myer's algorithms to output the path corresponding to the edit distance using  $\mathcal{O}(nD)$  time and  $\mathcal{O}(n)$  space, where  $D = d(s, t)$  and  $n = |s| + |t|$ .
- (SPOJ) 3. Let  $d \in \{1, 2, \dots, |s|\}$  be a period of a word  $s$  iff  $s[i] = s[i + d]$  whenever both  $s[i]$  and  $s[i + d]$  are defined, i.e.,  $i = 1, 2, \dots, |s| - d$ . You are given a word  $s$ . Print all periods of this word in decreasing order.
- (SPOJ) 4. Extra credit: we say that a sequence of numbers  $x_1, x_2, \dots, x_k$  is zigzag if no three of its consecutive elements create a nonincreasing or nondecreasing sequence. More precisely, for all  $i = 1, 2, \dots, k - 2$  either  $x_{i+1} < x_i, x_{i+2}$  or  $x_{i+1} > x_i, x_{i+1}$ . You are given two sequences of numbers  $a_1, a_2, \dots, a_n$  and  $b_1, b_2, \dots, b_m$ . The problem is to compute the length of their longest common zigzag subsequence.