- This problemset has *four* questions.
- For other questions, either send the solutions to gawry1+aos@gmail.com, or leave them in the envelope attached to the doors of my office (room 321).

1. Show that deciding whether a *system* of word equations is satisfiable is NP-hard, even if one side of each equation is constant.

2. Consider a word w over an alphabet Σ such that none of its two consecutive letters are identical. Show that there exist a partition of Σ into two disjoint sets Σ_ℓ and Σ_r such that there are at least $\frac{|w|-1}{4}$ appearances of pairs from $\Sigma_\ell \Sigma_r$ in w.

3. Consider a word-equation over one-variable (but perhaps many appearances of it) and its solution in a^*.
   (a) show that for each a: either there are no solutions from a^*, there is a unique such solution or each a^k is a solution of this equation;
   (b) devise a linear-time algorithm which, given a letter a and an equation, decides which of those three cases holds.

(SPOJ) 4. For extra credit: solve Morphing is fun problem. A longer version of the statement with a story is available there, and a short version is: given an alphabet Σ of size at most 26 and a morphism $f : \Sigma \to \Sigma^+$, consider the sequence of words a, $f(a)$, $f(f(a))$, $f(f(f(a)))$, ... Such a sequence **converges** if for any $k = 0, 1, 2, \ldots$ the k-th letter of $f^i(a)$ eventually stabilizes, which means that the k-th letter of all $f^i(a)$ is the same, for $i = i_0, i_0 + 1, i_0 + 2, \ldots$ (or there is no k-th letter in all these words). Given a description of f, check if the corresponding sequence stabilizes.