Assignment 5 for
Approximation Algorithms and Hardness of Approximation
Discussion:
Thursday, 26 June 2014, 14 pm

Assignment 1 (Baker’s shifting scheme for Vertex Cover on planar graphs)
Develop a PTAS for Vertex Cover in planar graphs. Use the following approach:

1. Give a dynamic program to solve Vertex Cover on graphs with treewidth at most \( w \) in time \( O(2^w n) \).
2. Show how to use this to find, given a planar graph, a vertex cover of size at most \((1 + \varepsilon)OPT\) in time \( 2^{O(1/\varepsilon)} n \).

Assignment 2 (Dominating Set and Hamiltonian Cycle on bounded tree-width graphs)
Show how to compute the minimum dominating set and a Hamiltonian Cycle on graphs of bounded tree-width. Can you find a PTAS for minimum dominating set on planar graphs using Baker’s shifting scheme?

Assignment 3 (PTAS for Euclidean \( k \)-TSP)
Given \( n \) points in the plane, the task is to find a minimum-length tour that visits at least \( k \) points. Give a PTAS for this problem.

Assignment 4 (Treewidth of \( k \)-outerplanar graphs)
In the lecture, we discussed that every \( k \)-outerplanar graph has treewidth at most \( 3k + 1 \). Let \( G \) be a \( k \)-outerplanar graph. Fill in the details for the approach that was discussed:

Part 2.1 Show how to transform \( G \) into a \( k \)-outerplanar graph \( G' \) of maximum degree 3 such that, given a tree-decomposition of \( G' \), you can construct a tree-decomposition of \( G \) of at most the same width. This proves \( tw(G) \leq tw(G') \).

Part 2.2 Let \( F \) be the edges of a spanning forest \( F \) of \( G' = (V, E) \). Hence, adding any edge \( e \in E \setminus F \) to \( F \) would create a cycle. For each such \( e \), we call the corresponding cycle fundamental cycle. The maximum load of a vertex \( v \in V \) is the number of fundamental cycles which contain \( v \). Similarly, the maximum load of an edge \( e \in F \) is the number of fundamental cycles which contain \( e \).

Prove that every \( k \)-outerplanar graph \( G' \) of maximum degree 3 has a spanning forest \( F = (V, F) \) for which every vertex \( v \in V \) and every edge \( e \in F \) has maximum load at most three.
Part 2.3  Show how to transform a spanning forest $\mathcal{F}$ of maximum load $\ell$ to a tree-decomposition of width $\ell + 1$.

*Hint: Use $\mathcal{F}$ as the underlying tree of the tree-decomposition.*

Part 2.4  Use the previous parts to show how to compute a tree-decomposition of width $3k + 1$ efficiently. What is the running time of the algorithm?