Anna Adamaszek, Andreas Wiese
Tutorials: Marvin Künnemann

Summer 2014

Assignment 5 for Approximation Algorithms and Hardness of Approximation

Discussion:
Thursday, 26 June 2014, 14 pm

Assignment 1 *(Baker's shifting scheme for Vertex Cover on planar graphs)*

Develop a PTAS for Vertex Cover in planar graphs. Use the following approach:

1. Give a dynamic program to solve Vertex Cover on graphs with treewidth at most w in time $O(2^w n)$.
2. Show how to use this to find, given a planar graph, a vertex cover of size at most $(1+\varepsilon)$OPT in time $2^{O(1/\varepsilon)}n$.

Assignment 2 *(Dominating Set and Hamiltonian Cycle on bounded tree-width graphs)*

Show how to compute the minimum dominating set and a Hamiltonian Cycle on graphs of bounded tree-width. Can you find a PTAS for minimum dominating set on planar graphs using Baker's shifting scheme?

Assignment 3 *(PTAS for Euclidean k-TSP)*

Given n points in the plane, the task is to find a minimum-length tour that visits at least k points. Give a PTAS for this problem.

Assignment 4 *(Treewidth of k-outerplanar graphs)*

In the lecture, we discussed that every k-outerplanar graph has treewidth at most $3k+1$. Let G be a k-outerplanar graph. Fill in the details for the approach that was discussed:

Part 2.1 Show how to transform G into a k-outerplanar graph G' of maximum degree 3 such that, given a tree-decomposition of G', you can construct a tree-decomposition of G of at most the same width. This proves $\mathrm{tw}(G) \leq \mathrm{tw}(G')$.

Part 2.2 Let F be the edges of a spanning forest $\mathcal{F}$ of $G' = (V, E)$. Hence, adding any edge $e \in E \setminus F$ to $\mathcal{F}$ would create a cycle. For each such e, we call the corresponding cycle *fundamental cycle*. The *maximum load of a vertex* $v \in V$ is the number of fundamental cycles which contain v. Similarly, the *maximum load of an edge* $e \in F$ is the number of fundamental cycles which contain e.

Prove that every k-outerplanar graph G' of maximum degree 3 has a spanning forest $\mathcal{F} = (V, F)$ for which every vertex $v \in V$ and every edge $e \in F$ has maximum load at most three.

Hint: Use induction over k and think of deleting all edges R on the exterior face of an embedding of G'. Extend a spanning forest of the resulting interior a bit - how many faces of G' have a fundamental cycles of some $e \in R \setminus \mathcal{F}$ as boundary?

Part 2.3 Show how to transform a spanning forest $\mathcal{F}$ of maximum load ℓ to a tree-decomposition of width $\ell + 1$.

Hint: Use $\mathcal{F}$ as the underlying tree of the tree-decomposition.

Part 2.4 Use the previous parts to show how to compute a tree-decomposition of width $3k+1$ efficiently. What is the running time of the algorithm?