Assignment 6 for
Approximation Algorithms and Hardness of Approximation
Discussion:
Thursday, 3 July 2014, 14 pm

Assignment 1 (Iterative Rounding for minimum-cost perfect matching on bipartite graphs)

Let $G = (V, E)$ be a bipartite graph. In the lecture, we used the iterative rounding approach to the linear program

$$\text{maximize} \quad \sum_{e \in E} w_e x_e$$

subject to

$$\sum_{e \in \delta(v)} x_e \leq 1 \quad \forall v \in V$$

$$x_e \geq 0 \quad \forall e \in E.$$ 

Adapt this approach to the following LP and prove that minimum-cost perfect matching is polynomially solvable on bipartite graphs:

$$\text{minimize} \quad \sum_{e \in E} w_e x_e$$

subject to

$$\sum_{e \in \delta(v)} x_e = 1 \quad \forall v \in V$$

$$x_e \geq 0 \quad \forall e \in E.$$ 

Assignment 2 (Iterative Rounding for VC on bipartite graphs)

Given a bipartite graph $G = (V, E)$, show how to solve vertex cover optimally by using the iterative rounding approach. Consider the following LP.

$$\text{minimize} \quad \sum_{v \in V} c_v x_v$$

subject to

$$x_u + x_v \geq 1 \quad \forall e = \{u,v\} \in E$$

$$x_v \geq 0 \quad \forall v \in V.$$ 

Show that you can always find a vertex $v$ with $x_v = 0$ or $x_v = 1$ and recurse on an easier subproblem.

Hint: Delete isolated vertices. What properties on the extremal points can you derive from the rank?
Assignment 3 (Survivable Network Design)

In the lecture, the following claim was left unproven: Let $r_{uv} \in \mathbb{N}$ for $u, v \in V$ with $r_{uv} = r_{vu}$ be given and define $r(S) := \max_{u \in S \cup \partial S} r_{uv}$. Show that $r$ is a skew supermodular function, i.e., for any $S, T \subseteq V$, one of the following statement holds:

1. $r(S) + r(T) \leq r(S \cup T) + r(S \cap T)$, or,
2. $r(S) + r(T) \leq r(S \setminus T) + r(T \setminus S)$. 