

# Introduction to lower bounds

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16.06.2014

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- We will learn techniques on how to prove lower bounds in various models, mainly decision trees, pointer machine, cell probe and indexability models.
- We will then move on to a detailed study of some external-memory data structures, and show how to derive lower bounds in this recent model.

- Warm-up
  - Sorting lower bounds in the decision tree model.
  - Convex hull lower bound.
- Comparison trees- maximum-element.
- Linear decision trees(Dobkin and Lipton).
- Algebraic decision trees(Steele and Yao).
- Algebraic computation trees. Ben-Or's theorem.

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- **3SUM:** Do any three elements in the input set  $\{x_1, \dots, x_n\}$  sum to zero?