Universität des Saarlandes
FR 6.2 Informatik

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Excercises
Online Algorithms

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss14/OnlineAlgos/

Sheet 1 Deadline: 1.05.2014

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam.

Exercise 1 (*10 points*) It can be shown, that no deterministic online algorithm for the cow-path problem can obtain a competitive ratio strictly less than 9. Your task is to prove a (possibly weaker) lower bound ℓ on the competitive ratio that can be obtained by any deterministic online algorithm for the cow-path problem.

Hint: An algorithm can be described by a sequence of turns. Let $f(i)$ be the number of steps that the cow makes between last crossing the origin and turn i.

Grading: Assuming that your proof is correct you will be assigned:

3 points,	if $3 \leq \ell < 5$,
10 points,	if $5 \leq \ell < 8.99$,
15 points$(10 + 5$ bonus points$)$,	if $\ell \geq 8.99$.

Exercise 2 (*10 points*)
Describe and analyze, a randomized online algorithm for the cow-path problem, that has a competitive ratio of at most 7.

Exercise 3 (*4+6 points*)
Prove that algorithms LIFO (Last In First Out) and LFU (Least Frequently Used), both have an unbounded competitive ratio for the paging problem.

Exercise 4 (*5+5 points*)
The *Online Bin Packing* problem: we have an infinite supply of unit-size bins, and items of sizes $a_1, a_2, \ldots a_n$ arrive over time (we may assume that for all i, $0 < a_i \leq 1$). The goal is to pack the items into bins, so that the capacity is not exceeded on any bin, and the number of used bins is minimized.

The *First Fit* algorithm for the online bin packing problem: Consider the bins in some fixed prespecified order, and once an item arrives pack it to the first bin it fits.

Prove that:

(i) First Fit has a competitive ratio of 2, and

(ii) First Fit has a competitive ratio of at least $3/2$, i.e., give a sequence of items for which First Fit uses at least $3/2$ times the number of bins of an optimal solution.