Exercise 1 (10 points)
Searching on a star is a generalization of the cow-path problem: There are $m$ rays all originating from the same point $s$. An agent starts at $s$ and is searching for some item located at some unknown distance on an (again unknown) ray. We consider the algorithm that visits the rays in a circular fashion, and walks a distance of

$$x_i = \left( \frac{m}{m-1} \right)^i$$

on the $i$'th ray it visits. Prove that the above algorithm, has a competitive ratio of

$$1 + 2 \left( \frac{m^m}{(m-1)^{m-1}} \right).$$

Exercise 2 (15 points)
We consider the problem of online scheduling with the objective of minimizing the total flow time $\sum_i (C_i - r_i)$ of the schedule. Algorithm rank round robin (RRR) schedules at every instantaneous time $t$ each unfinished task $i$ for an amount proportional to $rank_t(i)$, where $rank_t(i)$ denotes the number of tasks that are unfinished at $t$ and were released no later than $r_i$, i.e., $rank_t(i) := |\{j| r_j \leq r_i \& j \text{ is unfinished at } t\}|$.

So, if there are $k$ unfinished tasks at timepoint $t$, the $i$'th of them (ordered by release times) would be assigned an $i/\sum_{j=1}^{k} j$ fraction of the processor.

Prove that RRR is $(2 + \epsilon)$-speed $O(1)$-competitive.

Hint: Use the potential function

$$\Phi(t) := \sum_{i \in RRR(t)} z_i(t)rank_t(i),$$
where (as in the lecture), \( z_i(t) = \max\{p_i^{RRR}(t) - p_i^{OPT}(t), 0\} \), \( p_i^A(t) \) is the processing volume that is unfinished for task \( i \) under algorithm \( A \) at timepoint \( t \), and \( RRR(t) \) is the set of unfinished tasks under \( RRR \) at time \( t \).

**Exercise 3 (8+7 points)**
Recall the online scheduling problem for minimizing the total flow time \( \sum_i (C_i - r_i) \) on a single-processor from the lecture. The algorithm *Shortest Remaining Processing Time (SRPT)* processes at every point in time the task with the shortest remaining processing time among all unfinished tasks. Prove that:

a) SRPT achieves a competitive ratio of 1 for the objective of minimizing total flow-time on a single processor.  
*Hint: Try to use proof by contradiction, and apply an exchange argument.*

b) SRPT has a competitive ratio strictly greater than 1 for the objective of weighted flow time on a single processor. In this problem every task \( i \) is also associated with a weight \( w_i \) and we wish to minimize

\[
\sum_i w_i (C_i - r_i) .
\]