Excercises
Online Algorithms

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss14/OnlineAlgos/

Sheet 5 Deadline: 01.07.2014

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam.

Exercise 1 (4+8+8 points)
Consider the algorithm for fractional online set cover from the lecture and adjust it as follows. Upon arrival of element \( e \), we execute the same algorithm, but change the update of the primal variables \( x_S \) with \( e \in S \). Consider the following update rules:

\[
\begin{align*}
\text{a) } x_S &= x_S \left(1 + \frac{1}{c_S}\right) + \frac{1}{mc_S} \\
\text{b) } x_S &= x_S \left(1 + \frac{1}{c_S}\right) + \frac{1}{c_S} \\
\text{c) } x_S &= x_S + \frac{1}{dc_S}
\end{align*}
\]

What is the best competitive ratio in \( m \) or \( d \) you can show for each case?

Exercise 2 (8+6 points)
Consider an adjusted algorithm for the revenue optimization problem. Upon arrival of item \( j \) we assign it greedily to the buyer \( i \) that yields the maximum revenue (i.e., over all buyers, \( i \) maximizes the minimum of \( b_{ij} \) and his remaining budget).

\[
\begin{align*}
\text{a) Show that the competitive ratio of this algorithm cannot be better than 2. It should also show that the lower bound holds for any value of } R_{\text{max}} \leq 1. \\
\text{b) What is the best upper bound you can prove on the competitive ratio of this algorithm?}
\end{align*}
\]

Exercise 3 (6 points)
Consider the load balancing problem with identical machines, where the load of task \( j \) is the same on every machine \( i \), i.e., \( p_{ij} = p_j \geq 0 \). The greedy algorithm assigns a task upon arrival to the machine where the current load is minimum. Show that the greedy algorithm is 2-competitive.