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Deadline: lecture on May 4, 2015

Algorithmic Game Theory

Summer 2015

Exercise Set 1

Please hand in your solutions at the beginning of the lecture on May 4.
You may work in groups of up to three students.

Exercise 1: (5 + 3 Points)
Given the game with the following costs (lower left corner of a cell refers to the row player; upper right corner to the column player):

	A	B	C	D
A	6 4	5 6	4 5	5 6
B	6 5	5 1	1 2	5 5
C	6 6	3 5	3 3	2 1
D	6 7	3 7	5 6	7 7

a) What are the pure Nash equilibria? Explain why exactly those states are pure Nash equilibria and why no other state is one.

b) Why can no other strategy in this game be part of any mixed Nash equilibrium?
Or: What are the strategies chosen in a mixed Nash equilibrium and why?

Exercise 2: (4 Points)
The Hit-Below Game is given as follows. Each of n players announces a number in $\{1, \dots, k\}$. A prize of 1 Euro is split equally among the players whose chosen number is closest to 2/3 times the average number. Show that the Hit-Below Game has a unique Nash equilibrium and that in this equilibrium each player picks a pure strategy.

Exercise 3: (4 Points)
Consider the following four sets:

$$A = \{x \in \mathbb{R}^2 \mid x_1, x_2 \geq 0 \text{ and } x_1^2 + x_2^2 = 1\}$$
$$B = \{x \in \mathbb{R}^2 \mid x_1, x_2 \geq 0 \text{ and } 1 \leq x_1^2 + x_2^2 \leq 2\}$$
$$C = \{x \in \mathbb{R}^2 \mid x_1^2 + x_2^2 \leq 2\}$$
$$D = \{x \in \mathbb{R}^2 \mid x_1^2 + x_2^2 < 1\}$$

To which of those sets does Brouwer's fixed point theorem apply? For the other ones give a continuous function that has no fixed point.
Hint: In one dimension you need a closed set D because $f\colon (0,1] \to (0,1]$ with $f(x) = \frac{x}{2}$ does not have a fixed point. You need a bounded set because $f\colon [0,\infty) \to [0,\infty)$ with $f(x) = x+1$ does not have a fixed point.

Exercise 4: (4 Points)

Note: In the exercise, we consider *utilities* (or payoffs) rather than *costs*. Equilibria etc. are defined as if utilities were negative costs.

Consider the *single-item first-price auction game* with two players. Each player $i \in \{1, 2\}$ has strategies $[0, 1]$; his utility is parameterized by a value $v_i \in [0, 1]$. In state (b_1, b_2), the utilities are given as

$$u_1(b_1, b_2) = \begin{cases} v_1 - b_1 & \text{if } b_1 \geq b_2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad u_2(b_1, b_2) = \begin{cases} 0 & \text{if } b_1 \geq b_2 \\ v_2 - b_2 & \text{otherwise} \end{cases} .$$

Under what condition (regarding v_1 and v_2) is there a pure Nash equilibrium? State all pure Nash equilibria.