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Algorithmic Game Theory

Summer 2015

Exercise Set 3

Exercise 1: (1+1+1 Points)
Consider the game defined by this cost matrix:

	A	B
A	1 1	0 1
B	1 0	1000 1000

(a) List all pure Nash equilibria.

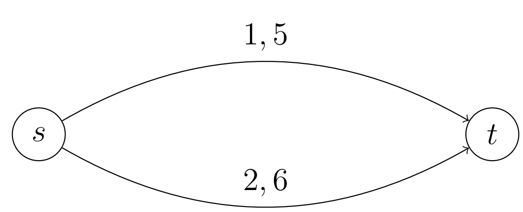
(b) Give a mixed Nash equilibrium that is not a pure Nash equilibrium.

(c) Give a coarse correlated equilibrium that is not a mixed Nash equilibrium.

Exercise 2: (3 Points)
Give an example of a sequence of cost vectors ℓ^t and strategy choices p^t such that the external regret is negative.

Exercise 3: (4 Points)
The no-regret algorithm analyzed in class was stated such that the overall length of the sequence T is given as a fixed parameter. Give a no-regret algorithm that works without such a parameter for all possible T. Use the algorithm from class as a subroutine (you do not need to analyze it again). Start with $T = 1$ as a guess and run the subroutine. Once the subroutine ends, restart it but double your guess.

Exercise 4: (1+2+1 Points)
Consider this symmetric network congestion game with two players:



(a) What are the price of anarchy and the price of stability for pure Nash equilibria?

(b) What are the price of anarchy and the price of stability for mixed Nash equilibria?
Hint: Start by listing all mixed Nash equilibria. To obtain these start with a sentence like, "*Let σ be a mixed Nash equilibrium with* $\sigma_1 = (\lambda_1, 1 - \lambda_1)$, $\sigma_2 = (\lambda_2, 1 - \lambda_2)$," and continue by deriving properties of λ_1 and λ_2.

(c) What is the best price-of-anarchy bound that can be shown via smoothness?

Exercise 5: (3 Points)

For every $M \geq 1$, give an example of a two-player network congestion game whose price of anarchy for pure Nash equilibria is at least M.

Exercise 6: (3 Points)

Fair cost sharing games are congestion games with delays $d_r(x) = c_r/x$ for some constant c_r for all $r \in \mathcal{R}$.

(a) Show that fair cost sharing games with n players are $(n, 0)$-smooth.

(b) For every n, give an example of an n-player fair cost sharing game whose price of anarchy for pure Nash equilibria is at least n.