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Algorithmic Game Theory

Summer 2015

Exercise Set 4

Exercise 1: (4 Points)
Prove that in every network cost-sharing game, the POA is at most k, where k is the number of players.

Exercise 2: (3+3+3 Points)
Arrow's theorem indicates that no social welfare function (SWF) over three or more outcomes is Pareto efficient (PE), independent of irrelevant alternatives (IIA) and not dictatorial.

(a) Design a SWF over three or more outcomes that is PE and not dictatorial.

(b) Design a SWF over three or more outcomes that is IIA and not dictatorial.

(b) Design a SWF over two outcomes that is PE, IIA and not dictatorial.

Please also explain why your designed SWF generates transitive preferences.

Exercise 3: (2+5 Points)
Recall the following simple majority rule: assume there are odd number of voters for simplicity, for each pair of outcomes o_1 and o_2, let $o_1 \succ_W o_2$ if a majority of voters prefer o_1 over o_2. We showed in class that by the Condorcet Paradox, the majority rule may produce non-transitive social preference even if all individual preferences are transitive.
Now let's focus on a special class of preferences called *single-peak preferences*: assume that the set of outcomes is the interval $[0, 1]$, and for each voter i, there exists an outcome o_i^*, such that if $o_i^* > o_1 > o_2$ or $o_2 > o_1 > o_i^*$, then $o_1 \succ_i o_2$.

(a) Provide an interpretation of single-peaked preferences.

(b) Assume that the number of voters is odd. Show that if all voters have single-peaked reference, then majority rule produces a social preference relation that is total and transitive.