Xiaohui Bei
Thomas Kesselheim

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Algorithmic Game Theory

Summer 2015

Exercise Set 6

Exercise 1: (7 Points)
Compute the virtual valuation function of the following distributions.

(a) The uniform distribution on $[a, b]$.

(b) The exponential distribution with rate $\lambda > 0$ (on $[0, \infty)$).

(c) The distribution given by $F(v) = 1 - \frac{1}{(v+1)^c}$ on $[0, \infty)$, where $c > 0$ is some constant.

Which of these distributions are regular (meaning the virtual valuation function is strictly increasing)?

Exercise 2: (7 Points)
Consider a single-item auction where bidder i's valuation is drawn from its own regular distribution F_i (i.e., the F_i's can be different).

(a) Give a formula for the winner's payment in an optimal auction, in terms of the bidders' virtual valuation functions.

(b) Show by example that, in an optimal auction, the highest bidder need not win, even if it has a positive virtual valuation. [Hint: two bidders with valuations from different uniform distributions suffices.]

(c) Give an intuitive explanation of why the property in (b) might be beneficial to the revenue of an auction.

Exercise 3: (6 Points)
Consider a single-item auction with n bidders with valuations drawn i.i.d. from a regular distribution F. Prove that the expected revenue of the Vickrey auction (with no reserve price) is at least $\frac{n-1}{n}$ times that of the optimal auction (with the same number n of bidders). [Hint: deduce this statement from the Bulow-Klemperer theorem. When one new bidder is added, how much can the maximum-possible expected revenue increase?]