Beyond classical circuit design
lecture 12

Metastability

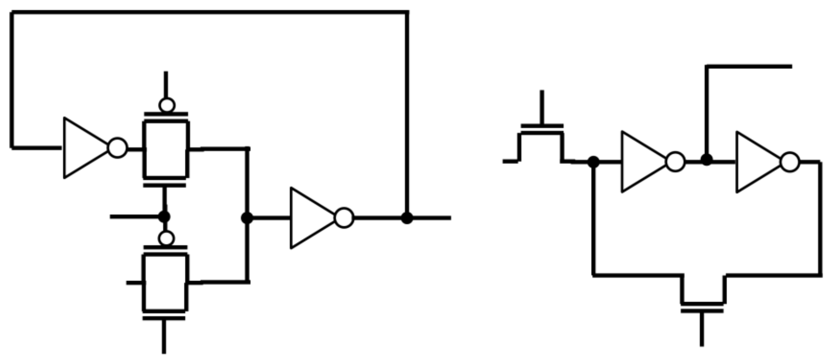
Metastability

storage element:

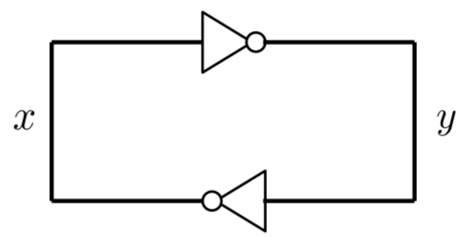
can store stable 0 & 1

potentially also stores a third metastable state

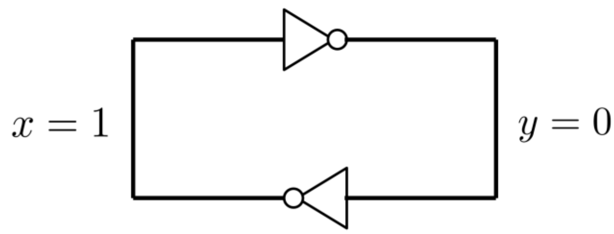
Remember: storage loops



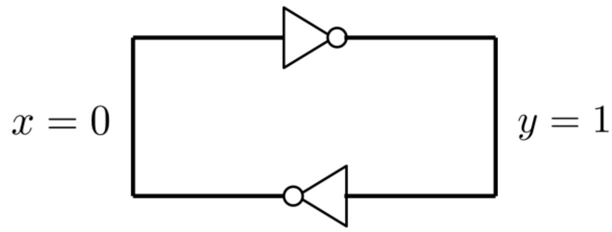
Storage loop



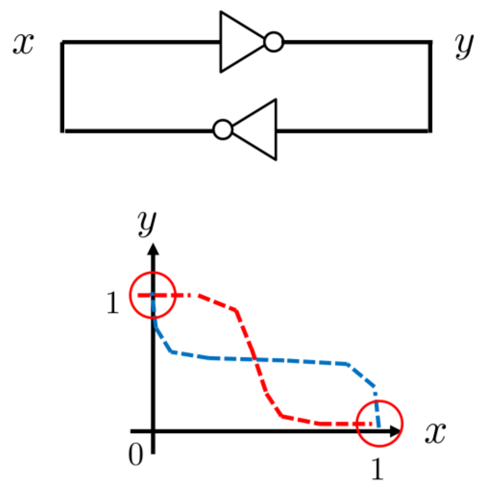
Stable output 0



Stable output 1

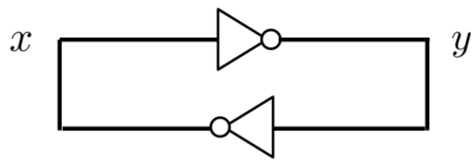


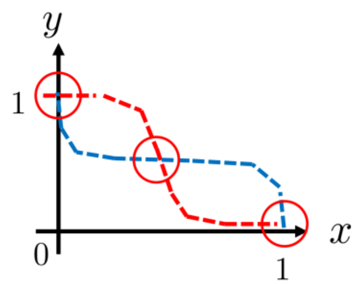
Stable states



below: input-output voltage after transient effects decayed

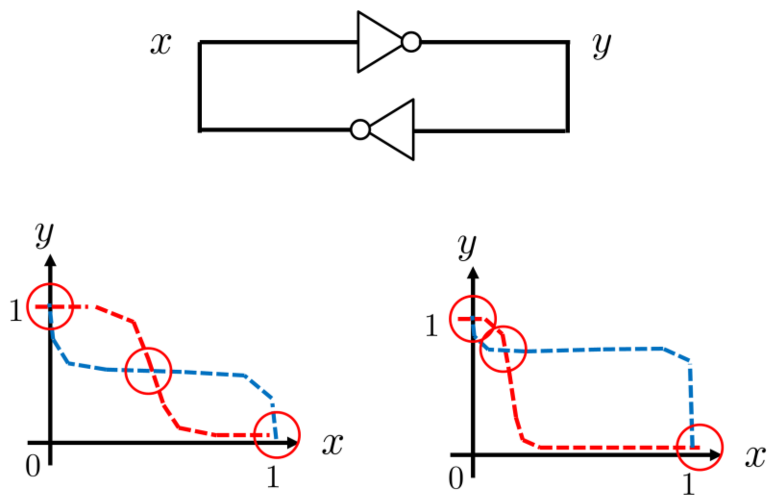
Mestastable state





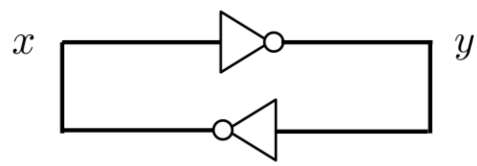
Third state also solves the steady state equations of both inverters.
However, “metastable”: small disturbance makes it resolve to a stable 0 or 1 state.

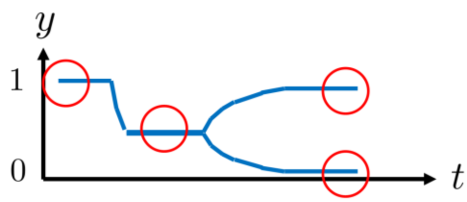
Mestastable state engineering

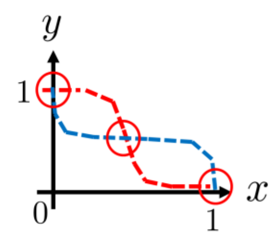


low & high threshold inverters to shift metastable state to a region that is well recognized as 1 (here) or 0 when read.

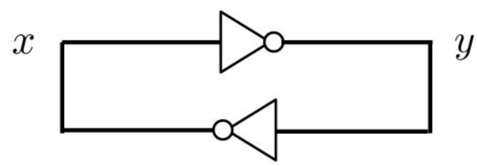
Transient behavior



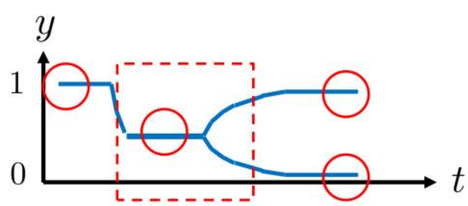


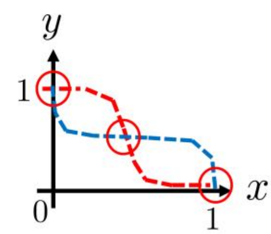


Propagation

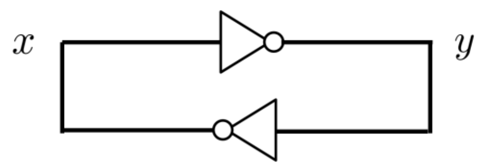


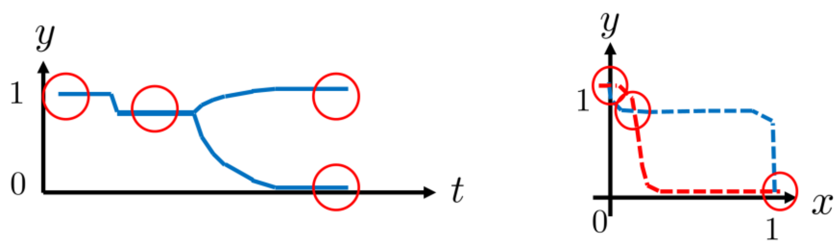
read during non-resolved time



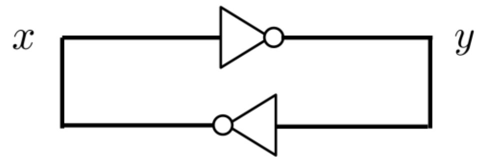


Transient behavior

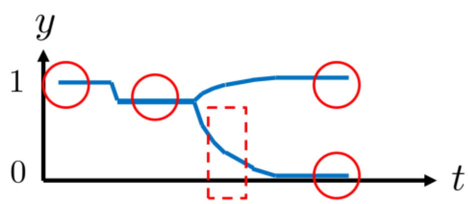


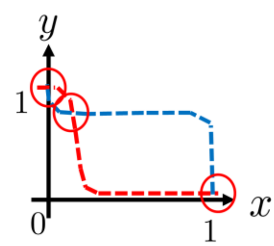


Propagation



read during transition time, if transition to 0





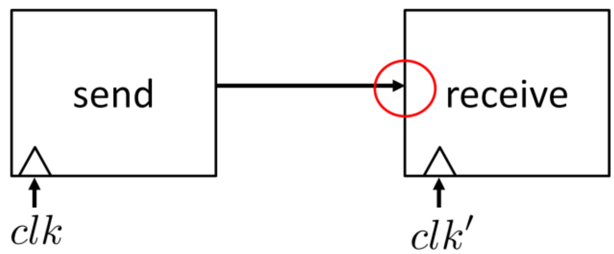
Metastability

causes:

- violation of setup/hold times
- induced by faults (e.g. particle hits)

Metastability

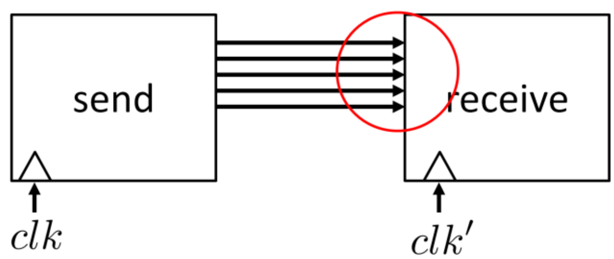
Clock domain crossings



Uncorrelated phase differences

Metastability

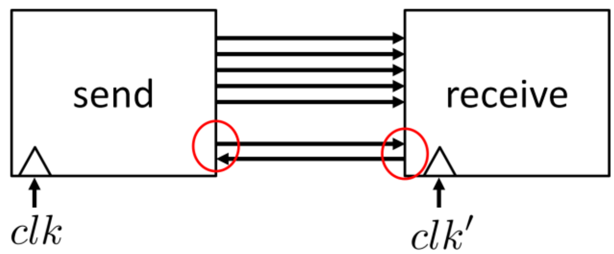
Clock domain crossings



multiple data rails

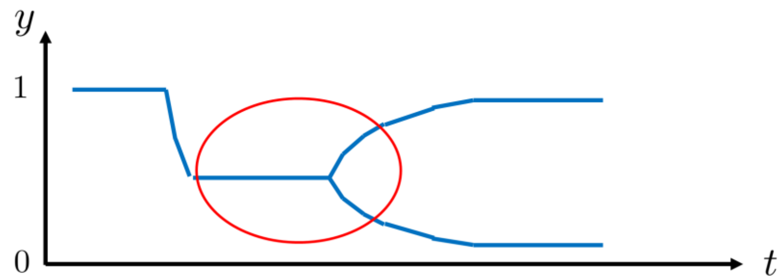
Metastability

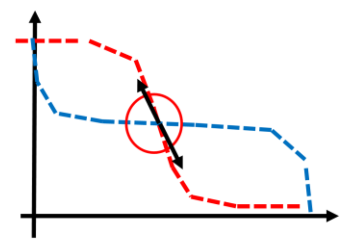
Clock domain crossings



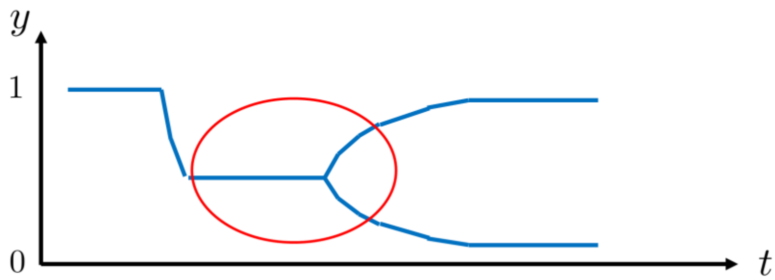
multiple data rails: handshaking -> correlation

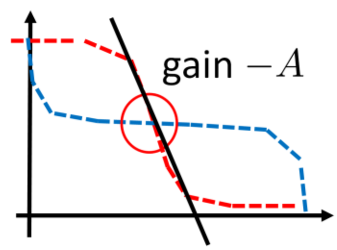
Resolving phase



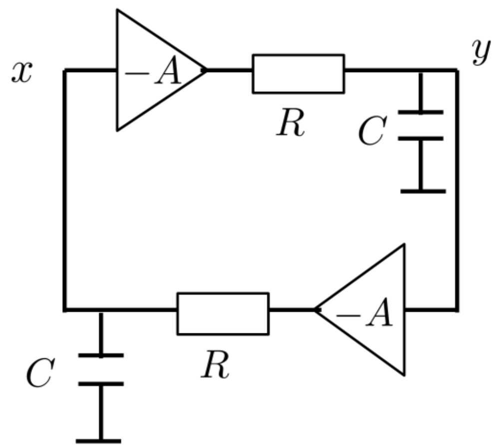


Resolving phase





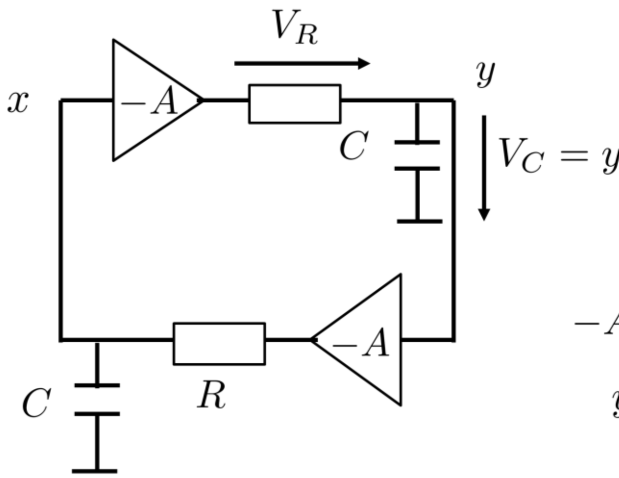
First order model



@ metastable point

gain –A, first order RC model.
Here: assume that both inverters same characteristics

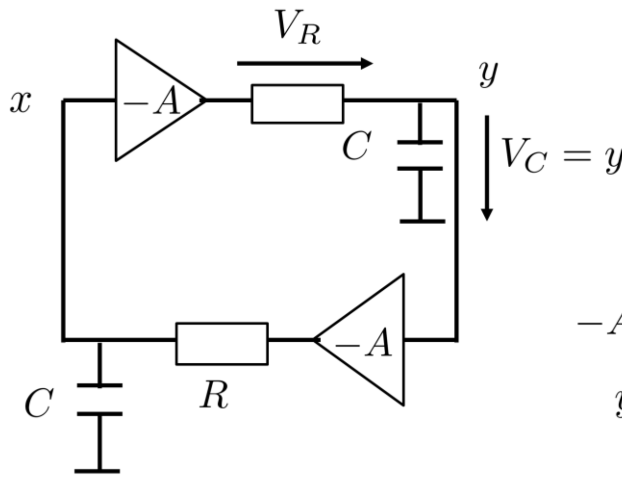
First order model



$$-Ax = V_R + V_C$$

$$y = -Ax - V_R$$

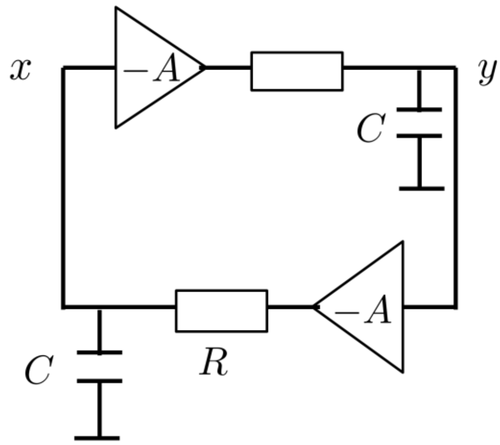
First order model



$$-Ax = V_R + V_C$$

$$y = -Ax - V_R$$

First order model



$$y = -Ax - RC\frac{dy}{dt}$$

$$x = -Ay - RC\frac{dx}{dt}$$

First order model

$$y(t) = -Ax(t) - RC\frac{dy(t)}{dt}$$
$$x(t) = -Ay(t) - RC\frac{dx(t)}{dt}$$

Laplace →

$$Y(s) = -AX(s) - RC(sY(s) - y(0))$$
$$X(s) = -AY(s) - RC(sX(s) - x(0))$$

First order model

$$Y(s) = -AX(s) - RC(sY(s) - y(0))$$

$$X(s) = -AY(s) - RC(sX(s) - x(0)) \longrightarrow$$

$$Y = \frac{A^2}{(1+s\tau)^2} Y - \frac{A\tau x(0)}{(1+s\tau)^2} + \frac{\tau y(0)}{1+s\tau} \longrightarrow$$

$$Y(1 - \frac{A^2}{(1+s\tau)^2}) = -\frac{A\tau x(0)}{(1+s\tau)^2} + \frac{\tau y(0)}{1+s\tau}$$

tau = RC

First order model

$$Y(1 - \frac{A^2}{(1+s\tau)^2}) = -\frac{A\tau x(0)}{(1+s\tau)^2} + \frac{\tau y(0)}{1+s\tau} \longrightarrow$$

$$Y = \frac{1}{2}\left(\frac{y(0)-x(0)}{-\left(\frac{A-1}{\tau}+s\right)} + \frac{y(0)+x(0)}{-\left(-\frac{A+1}{\tau}+s\right)}\right) \xrightarrow{\text{inv Laplace}}$$

$$y = \frac{1}{2}\left((y(0) - x(0))e^{\frac{A-1}{\tau}t} + (y(0) + x(0))e^{-\frac{A+1}{\tau}t}\right)$$

First order model

$$Y(1 - \frac{A^2}{(1+s\tau)^2}) = -\frac{A\tau x(0)}{(1+s\tau)^2} + \frac{\tau y(0)}{1+s\tau} \longrightarrow$$

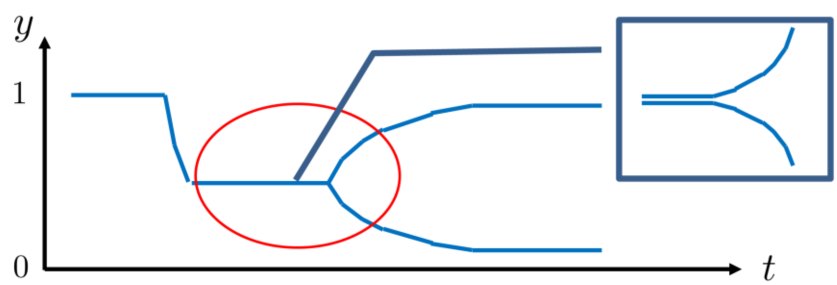
$$Y = \frac{1}{2}\left(\frac{y(0)-x(0)}{-(\frac{A-1}{\tau}+s)} + \frac{y(0)+x(0)}{-(-\frac{A+1}{\tau}+s)}\right) \xrightarrow{\text{inv Laplace}}$$

$$y = \frac{1}{2}\left((y(0) - x(0))e^{\frac{A-1}{\tau}t} + (y(0) + x(0))e^{-\frac{A+1}{\tau}t}\right)$$

...dominant term

Resolving

$$y = \frac{1}{2}\left((y(0) - x(0))e^{\frac{A-1}{\tau}t}\right)$$

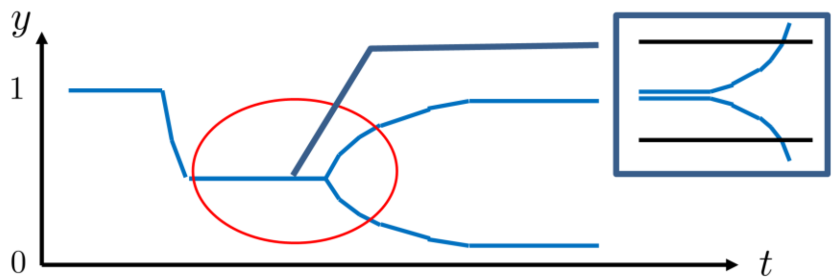


mind: the above equation only holds near the metastability point.
it assumes e.g. that gain A is constant and not depenent on the input voltages x or y of the inverters.
gain is different (around 0) for points near stable 0 or 1.

Resolving

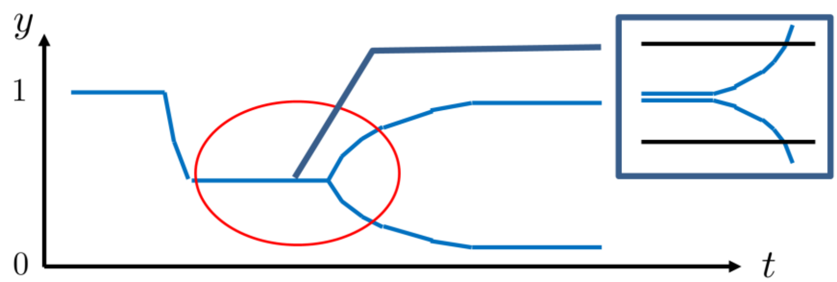
If y crossed some boundaries -> resolves quickly

How long does it need to reach those?



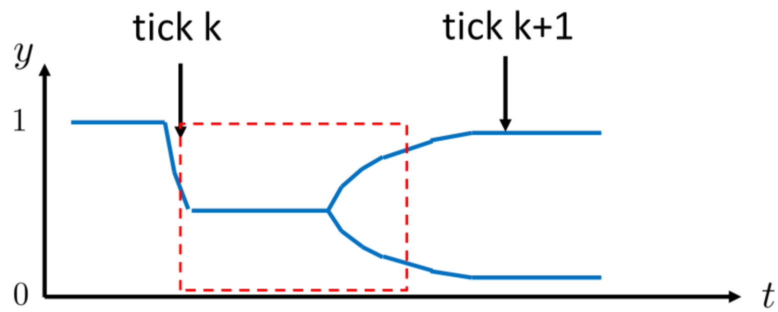
Resolving

$$\ln\left(\frac{2B}{|y(0)-x(0)|}\right)\frac{\tau}{A-1} = t$$



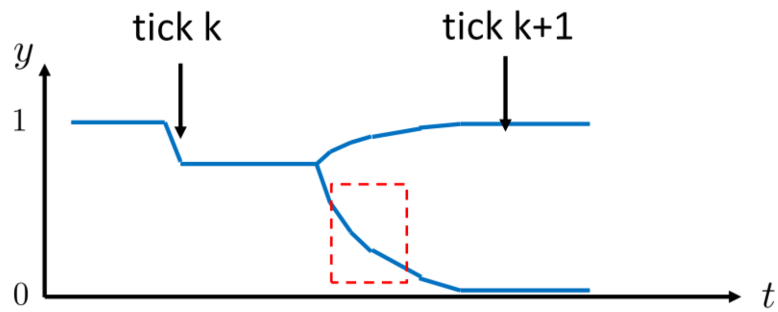
Resolving (1)

$$\ln\left(\frac{2B}{|y(0)-x(0)|}\right)\frac{\tau}{A-1} \leq T_{clk} - T_{offset}$$



Resolving (2)

$$\ln\left(\frac{2B}{|y(0)-x(0)|}\right)\frac{\tau}{A-1} \in T_{clk} - T_{offset} + [-\Delta, \Delta]$$



Resolving

$$y(0) - x(0)$$

Depends on phase relation of input change to clock transition.

Several model assumptions to quantify it.

Literature: typically exponentially/uniformly distributed input changes & linear change over time.

MTBU

upset [here, (1)]: if metastability resolves **after** next tick

$$\mathrm{MTBU} \propto e^{\frac{A-1}{\tau}(T_{clk} - T_{offset})}$$

increase T_{clk} by stacking flip-flops
-> synchronizer chains

meantime between upset: expected meantime between two upsets (= not resolved at next clock tick)

MTBU

upset [alternative, (2)]: if metastability resolves **around** next tick

better, but still:

$$\text{MTBU} \propto e^{\frac{A-1}{\tau}(T_{clk}-T_{offset})}$$

Impossibility Result

Marino: “General theory of metastable operation”, 1981.

Thm. Any bistable element must have executions with arbitrarily long delays until a stable 0 or 1 state is reached.

Impossibility Result

System model: differential equation on continuous system space.

Proof idea: topological.

- 0 & 1 bounded time attractors disconnected regions in system space.
- executions: continuous traces in system space.

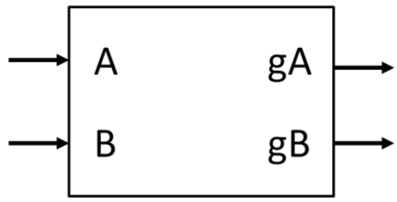
-> must cross non-attractor region

Impossibility Result

Implies impossibility of metastability-free bistable elements by reduction:

arbiter, inertial delay, flip-flop, latch, C-Element, etc.

(One-shot) Arbiter



Initially: all 0.

Input: at most one 0-1 transition at A or B or both.

Arbiter Properties

Mutex. Either gA makes 0-1 transition or gB but never both.

Bounded Time: Output transitions occur at most T1 time after the first input transition.

Validity. If A/B transition at least T2 time before B/A transition -> gA/gB transition occurs