Beyond classical chip design
lecture 2

Self-stabilization (continued)

Further Reading

Dijkstra, Edsger W.: *Self-stabilization in spite of distributed control.* Selected writings on computing: a personal perspective. Springer New York, 1982. 41-46.

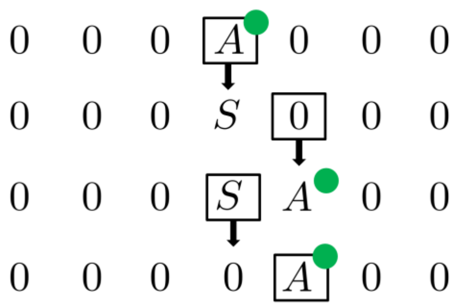
Brown, Geoffrey M., Mohamed G. Gouda, and Chuan-Lin Wu: *Token systems that self-stabilize.* Computers, IEEE Transactions on 38.6 (1989): 845-852.

What we had...

Algorithm:

... enabled = non-trivial transition

... token



What we wanted...

Algorithm:

3 states,

uniform,

very simple predicate and transition function

-> 

But ~~wait...~~

... $s(t) \in [n]$

Generally...

Stable transition functions:

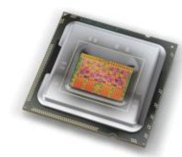
i can make a transition to c at time t &

i **cannot** make a transition to c at time $t+1$

->

i made a transition at time $t+1$

(and thus is in c at time $t+1$)



“i can make a transition to c at time t” defined as:
i is not in state c at time t and delta_i(x_{i-1}(t) , x_i(t) , x_{i+1}(t)) = c

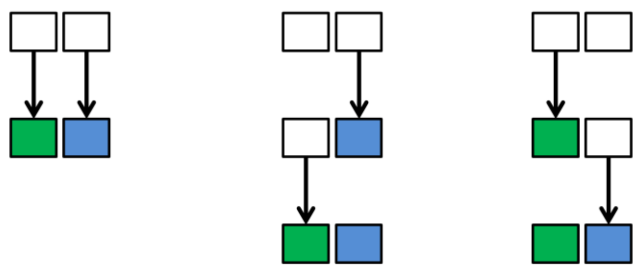
-> “i cannot make a transition to c at time t” defined as:
i is either in state c at time t or delta_i(x_{i-1}(t) , x_i(t) , x_{i+1}(t)) \neq c

Generally...

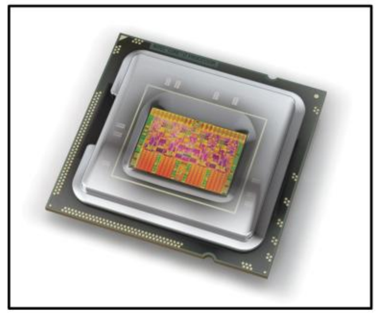
"distributed schedule" $s(t) \subseteq [n]$

stable + distributed schedule ->

"linearizable to" schedule [later]

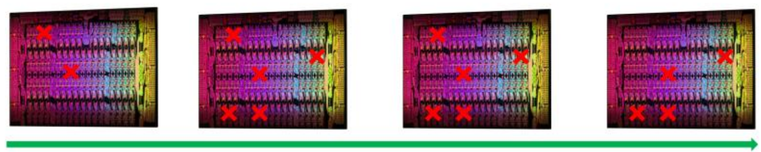


Reliable designs

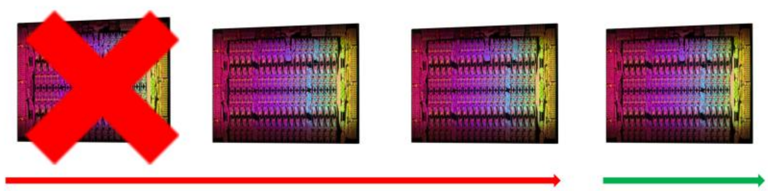


Reliable designs

Fault-tolerance.



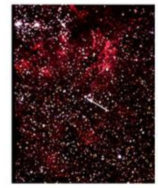
Self-stabilization.



Robustness...

$0 \quad 0 \quad 0 \quad \boxed{A} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

Robustness...



$0 \quad 0 \quad 0 \quad \boxed{A} \quad 0 \quad 0 \quad 0 \quad \boxed{A} \quad 0$

... no mutex.

Fault -> state flip -> a new token

Self-stabilization

For all initial states, all executions from this state: exists a time T:

T-postfix fulfils requirements.

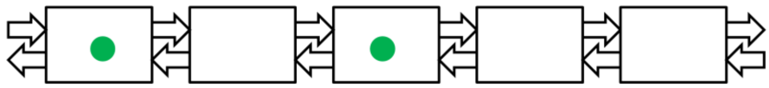
Exists a time T: for all initial states, all executions from this state:

T-postfix fulfils requirements.

difference between self-stabilization and bounded self-stabilization
T: only makes sense in stronger schedulers than weak-fair schedulers

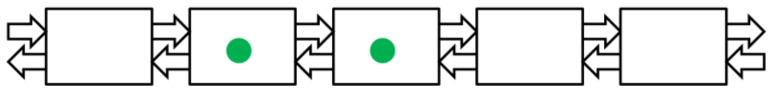
Example problem

Token passing system.



Example problem

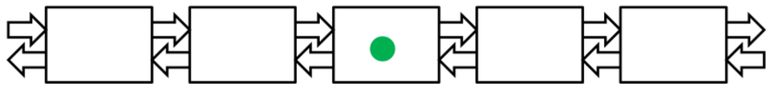
Token passing system.



Tokens see each other.

Example problem

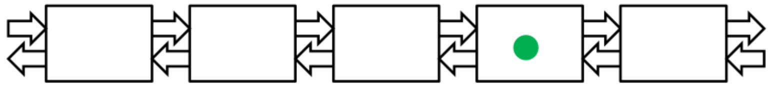
Token passing system.



Tokens merge.

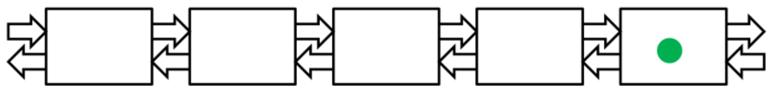
Example problem

Token passing system.



Example problem

Token passing system.



Self-stabilization

"equal speeds are bad"

Self-stabilization

"equal speeds are bad"

Solution 1. Randomness.

- implementation
- fault-free behavior

random solution: e.g. in synchronous scheduler.

Self-stabilization

“equal speeds are bad”

Solution 1. Randomness.

- implementation
- fault-free behavior

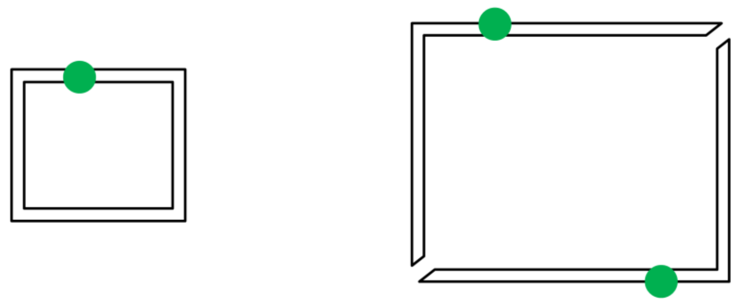
But: ... no token case!

Self-stabilization

Uniform deterministic solutions for all ring sizes?

Self-stabilization

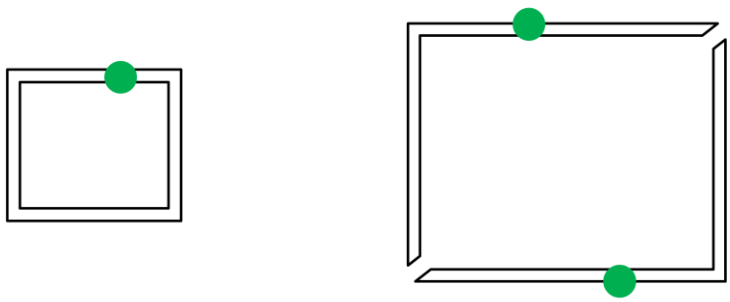
Uniform deterministic solutions for all ring sizes?



Proof that we cannot: duplicate ring. For a node i in the original ring there are corresponding nodes i' and i'' in the duplicated ring with the same initial state. Whenever i is scheduled, schedule i' and i''. Corresponding nodes i and i' and i'' will always have the same state (proof by induction). Thus there will be at least 2 tokens in the duplicated ring.

Self-stabilization

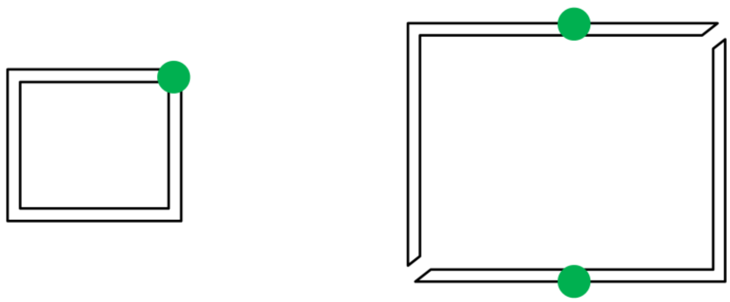
Uniform deterministic solutions for all ring sizes?



scheduling i‘ and i‘‘ when i is scheduled.

Self-stabilization

Uniform deterministic solutions for all ring sizes?



Double -> contradiction

Self-stabilization

Solution 2. Dijkstra (det, non-uniform, uses size)

machine 0:

if $x_{i-1} = x_i$ then $x'_i = x_i + 1 \mod (N+1)$

all others (1..N):

if $x_{i-1} \neq x_i$ then $x'_i = x_{i-1}$

N+1 nodes, node 0 with different code than others.

Self-stabilization

N “other“ nodes

N+1 states from $V = [N+1] = \{0, \ldots, N\}$

-> say, state N does not occur.

Self-stabilization

Obs 1. node 0 first one to have N.

Obs 2. from N(non-N)....(non-N) eventually reach N...N.

Obs 3. from N...N only 1 execution with mutex & weak fairness.

Self-stabilization

Prop 1. Show $\exists t : x_0(t) = N$

Assume not.

-> 0 makes bounded # non-trivial steps

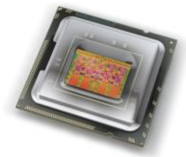
-> last at time t' with $x_0(t') = a$

-> eventually $a \ldots a$

-> eventually 0 makes step

-> contr.

Self-stabilization

 -> 

$V = [N]$: “mod N” instead of “mod N+1”?

Not with distributed scheduler. [hw]

Self-stabilization

not stable, but:

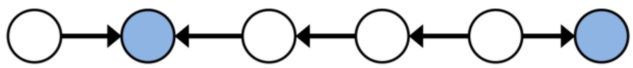
- works with distributed scheduler
- for all ring sizes exists solution

Self-stabilization

Solution 3. [Brown, Gouda]

not stable <-> two neighbours try to make a step at the same time

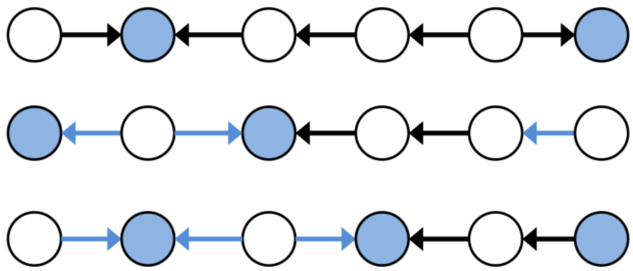
Self-stabilization



Prop 1. neighbour-mutex holds.

Self-stabilization

link-reversal e.g. full/partial reversal



...

Self-stabilization

Ring cut...

Prop 2. No deadlock. [hw]

Prop 3. Weak fairness. [hw]

What we obtain...

... link reversal gives a neighbour-mutex, weak fair scheduler.

potentially unstable algorithm

LR

distributed scheduler

Simulating scheduler

Distributed, weak-fair scheduler ->

Distributed, **neighbour-mutex**, weak fair scheduler.