

# Beyond classical chip design

## Exercise I

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**Ex 1.** A weakly fair distributed scheduler is a scheduler where all nodes make infinitely many steps and several nodes can make a step at the same time. A probabilistic distributed scheduler is a scheduler where each node has constant probability  $p > 0$  to be scheduled at a time  $t$ . Assume independence between these events. Prove that a distributed probabilistic scheduler is a weakly fair distributed scheduler with probability 1.

**Ex 2.** Let an adaptive probabilistic distributed scheduler be a scheduler for which the probability that a node scheduled at time  $t$  is  $p(t) > 0$ . Is an adaptive probabilistic distributed scheduler a weakly fair distributed scheduler with probability 1?

**Ex 3.** Prove the correctness of the token passing algorithm with states  $\{0, A, S\}$  by (i) proving mutex and (ii) proving weak fairness. Precisely state the assumptions made, induction hypothesis, etc.

**Ex 4.** Are these properties safety or liveness properties? Explain why.

- i) For all nodes  $i \in [n]$ , for all times  $t \geq 0$ , there is a time  $t' > t$  such that if node  $i$  sets its state to  $A$  at time  $t$ , then node  $i + 1$  sets its state to  $A$  at time  $t'$ .
- ii) For all nodes  $i \in [n]$ , for all times  $t \geq 0$ , if node  $i$  sets its state to  $A$  at time  $t$ , then node  $i + 1$  sets its state to  $A$  at time  $t + 2$ .
- iii) For all nodes  $i \in [n]$ , for all times  $t \geq 0$ , if node  $i$  sets its state to  $A$  at time  $t$ , then node  $i + 1$  sets its state to  $A$  a time  $t' \in [t, t^5]$ .

**Ex 5\*.** The token passing algorithm uses 2 bits to encode the state of each node. Is it possible to modify the algorithm such that it uses only two states and thus only one bit to encode the state? Either prove that it is not possible, or prove that it is possible by stating and proving correct an algorithm. You may also be able to solve it only for a subclass of all rings, e.g., only for rings of certain sizes, and not for the others.