Beyond classical chip design
Exercise IV

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Ex 16. In the slides we mentioned two definitions of the C-Element: the one from the two production rules (C1):

$$(a \wedge b) \to z\uparrow$$
$$(\neg a \wedge \neg b) \to z\downarrow$$

where initially $a = b = z = 0$; and the sequential CHP (C2):

$$[*[a \wedge b]; z\uparrow; [\neg a \wedge \neg b]; z\downarrow]$$

Are they equivalent in the sense that the circuit definied by (C1) implements the circuit definied by (C2) and vice versa, where the observable variables are $\{a, b, z\}$?

Ex 17. (2P) One can introduce the notion of an *environment*. In the slides we defined an environment to be a circuit. Intuitively an environment restricts the possible input scenarios, since it only allows executions of the environment circuit instead of all possible input scenarios.

Assume the following circuit (C3):

$$*[([a]|||[b]); z\uparrow; ([\neg a]|||[\neg b]); z\downarrow]$$

Prove implementation relations between (C3), (C1) and (C2) where $\mathcal{O} = \{a, b, z\}$.

Now assume the following environment:

$$z \to a\downarrow$$
$$\neg z \to a\uparrow$$
$$z \to b\downarrow$$
$$\neg z \to b\uparrow$$

where initially $a = b = z = 0$.

Again prove implementation relations between (C3), (C1) and (C2) but this time under the restrictions imposed by the environment; that means implementation relations between the compound circuits (C3)+Environment, (C1)+Environment, and (C2)+Environment.

Ex 18. (2P) Prove the following property on circuits: stable implies linearizable.

Think about and (briefly) discuss methods you think lead to stable circuits. Can you think of some circuits that are stable? For example ones that contain only Or gates, or ones that contain only NOT gates etc.