Motivation

Descartes’ Rule of Sign, Root Isolation and Möbius transformations.

Descartes’ Rule of Sign

• Let $f = \sum_{i=0}^{n} a_i x^i$ be a polynomial with $n$ real roots and $f(0) \neq 0$. Show that $a_i = 0$ implies that $a_{i-1} \cdot a_{i+1} < 0$.

• Determine isolating intervals for the real roots of

$$f = 30x^5 + 95x^4 - 2x^3 - 53x^2 - 4x + 6$$

by the use of the VCA algorithm.

Möbius Transformations

Show that, for $\lambda \in \mathbb{R}$ and $\mu \in \mathbb{R}\{0\}$, each of the following transformations

$$t_\lambda : z \rightarrow z + \lambda$$

$$h_\mu : z \rightarrow \mu z$$

$$r : z \rightarrow \frac{1}{z}$$

• is bijective on $\mathbb{C} := \mathbb{C} \cup \infty$.

• maps general circles to general circles. We call a subset $C \in \mathbb{C}$ in the complex space a general circle if $C$ is either a circle or a line with $\infty$.

• preserves angles, that is, given two curves $C_1$ and $C_2$ in $\mathbb{C}$ that intersect in a point $\xi$ with an included angle $\alpha$ and let $T$ be one of the transformations from above then the curves $T(C_1)$ and $T(C_2)$ intersect in $T(\xi)$ with an included angle $\alpha$ as well.

*Hint:* Consider two lines $L_i = \overline{a_i b_i}$, $i = 1, 2$, passing two distinct points $a_i, b_i \in C_i$. Compare the angle between the lines $L_i := \overrightarrow{T(a_i), T(b_i)}$ and that included by $L_1$ and $L_2$! What happens if $a_i$ and $b_i$ converge to $\xi$?

Have fun with the solution!