Homework 2

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This assignment is **due on October 29** in class. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be written individually without consulting someone else's solutions or any other source like the web.

Problem 1: consider the minimax theorem:

$$v^* = \min_{x \in S_m} \max_{y \in S_n} x^T A y = \max_{y \in S_n} \min_{x \in S_m} x^T A y, \tag{1}$$

where $S_l = \{x \in \mathbb{R}^l_+ : \sum_{i=1}^l x_i = 1\}$. Show that

$$v^* = \min\{v \mid x^T A \leq v e_n^T,\ x \in S_m\} = \max\{v \mid Ay \geq v e_m,\ y \in S_n\}.$$

Problem 2: (i) Show that $\min_{x \in S_m} \max_{y \in S_n} x^T A y \geq \max_{y \in S_n} \min_{x \in S_m} x^T A y$.

(ii) Let $A_1, \ldots, A_m$ be the rows of A and $A^1, \ldots, A^n$ be the columns of A. Show that a pair of strategies $x^* \in S_m$ and $y^* \in S_n$ are optimal if and only if for all i, j: $(x^*)^T A^j \leq A_i y^*$.

Problem 3: (i) Show that any matrix game (1) can be converted into an equivalent one in which each entry in the matrix A is in $[a, b]$, where $a, b \in \mathbb{R}$. Does the same reduction work if we are aiming at an ϵ-approximate saddle point?

(ii) Show that any matrix game (1) can be reduced to an equivalent one in which the matrix A is skew-symmetric. i.e., $A^T = -A$. What is the value of the reduced game?

Hint: Assume by using part (i) that the value of the game defined by A is in $(0, 1)$. Construct a game with matrix:

$$\begin{bmatrix} 0 & -A & e_m \\ A^T & 0 & -e_n \\ -e_m^T & e_n^T & 0 \end{bmatrix}.$$