Optimization II
Winter 2009/10

Homework 7

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This assignment is **due on Jan 18** in class. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Problem 1: Let $a_1, \ldots, a_n$ be vectors in R^m. Let $U = \{1, \ldots, n\}$ and $\mathcal{I}$ be those subsets S of U such that the vectors indexed by S (that is, $\{a_i \mid i \in S\}$) are linearly independent. Prove that $(U, \mathcal{I})$ is matroid.

Problem 2: Let $G = (V, E)$ be a bipartite graph with partitions L and R. Consider the following two matroids defined on E:

$$\mathcal{I}_1 = \{I \subset E \mid \deg_I(u) \leq 1 \text{ for all } u \in L\} . \quad (1)$$

$$\mathcal{I}_2 = \{I \subset E \mid \deg_I(u) \leq 1 \text{ for all } u \in L\} . \quad (2)$$

Show that $(E, \mathcal{I}_1)$ and $(E, \mathcal{I}_2)$ are indeed matroids. Prove that if a set of edges is independent in both matroids then it is matching.

Problem 3: Let $(U, \mathcal{I})$ be a matroid, and x be an **integral** solution of its polytope, $\left\{x \in R_+^{|U|} \mid x(S) \leq r(S)\ \forall S\right\}$. Show that $\{u \in U \mid x_u = 1\}$ belongs to $\mathcal{I}$.