Optimization II
Winter 2009/10

Homework 8

Khaled Elbassioni
Julián Mestre

This assignment is **due on Jan 25** in class. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Problem 1: The pair $(U, \mathcal{I})$ is called a *partition* matroid if there exists a partition $X_1, X_2, \ldots, X_l$ of U and numbers $k_1, k_2, \ldots, k_l$ such that $S \subseteq U$ is independent (i.e., $S \in \mathcal{I}$) if and only if $|X_i \cap S| \leq k_i$ for all i. Prove that the matroid exchange property indeed holds for such a subset system.

Problem 2: A rooted out-branching is a directed acyclic graph having a path from the root to every other vertex. In the minimum out-branching problem we are given a **directed** graph $D = (U, A)$, a root r, and weights $w : A \to R$. The objective is to find a minimum weight out-branching rooted at r. Show how to cast this problem as a matroid intersection problem.

Problem 3: Let $\mathcal{L}$ be a laminar family on a ground set with n elements. Prove that $|\mathcal{L}| \leq 2n - 1$. (Notice that leaf nodes are allowed to be singletons.) Provide an example showing that this bound is best possible.