Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Optimization II

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws10/AO/

Assignment 4 Deadline: We 15.12.2009

Notation A linear program is in standard form if it is written as $\min\{c^T x | Ax \leqslant b, x \geqslant 0\}$.

Exercise 1 *Standard form*

Write the following LP in standard form.

$$\begin{array}{rrrrcl} \min & 3x_1 & -5x_2 & +x_3 & & \\ \text{subject to} & x_1 & & +x_3 & = & -5 \\ & -3x_1 & +6x_2 & & \geqslant & 10 \\ & x_1 & -x_2 & -4x_3 & \leqslant & 7 \\ & x_1 & & & \geqslant & 0 \\ & & x_2 & & \leqslant & 0 \end{array}$$

Exercise 2 *Police scheduling*

The Saarbrücken police force has the following minimum daily requirements for policemen on duty.

Period	Minimum
0.00 – 4.00	15
4.00 – 8.00	35
8.00 – 12.00	65
12.00 – 16.00	80
16.00 – 20.00	40
20.00 – 24.00	25

Each policeman comes on duty at 0.00, 4.00, 8.00, 12.00, 16.00 or 20.00 hrs and works for eight consecutive hours.

(a) Formulate the problem of finding the duty schedule that minimizes the total number of policemen required. Assume that the same schedule is repeated day after day. You do not need to solve the problem.

(b) Give the dual of the LP under (a).

Exercise 3 *Bin packing*

Write a linear program to determine a lower bound for online bin packing algorithms based on the input sequence $1/7 + \varepsilon, 1/3 + \varepsilon, 1/2 + \varepsilon$. You do not need to determine the set of patterns, but only give the structure of the LP. To check that the competitive ratio is maintained in any given phase of the input, you must count only bins that have been opened in that phase or earlier; to do this, you will have to assign a *type* to each pattern which can be used to indicate the phase where bins with this pattern start to get used.
The LP does not have to be in standard form.

Exercise 4 *List scheduling*

Show that the algorithm List Scheduling is exactly $(2 - 1/m)$-competitive. For this, you will have to use two different lower bounds on the optimal makespan for a given input.