

## Problem Set 1

Due: Nov. 19

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**Note:** Please send the solutions to `hsun@mpi-inf.mpg.de` before the deadline.

**Problem 1** For any constant  $A > 0$ , show that any  $n$ -vertex  $A$ -expander has diameter  $O(\log n)$ .

**Problem 2** Let  $G$  be a  $d$ -regular  $n$ -vertex graph with spectral expansion  $\lambda$ . The normalized adjacency matrix of  $G$  is expressed by  $M$ . Prove that for any probability distribution  $\pi$ ,

$$\text{CP}(M\pi) - 1/n \leq \lambda^2 (\text{CP}(\pi) - 1/n).$$

**Problem 3** Let  $G$  be a  $d$ -regular graph where the eigenvalues of  $G$ 's Laplacian matrix are  $\rho_1 \leq \dots \leq \rho_n$ . Prove the following results:

1.  $\rho_n \geq \frac{nd}{n-1}$ .
2.  $\sum_{i=1}^n \rho_i^2 = nd(d+1)$ .

**Problem 4** Show that the distance of any linear code equals the minimum Hamming weight of a non-zero codeword.