Expander Graphs in Computer Science **WS 2010/2011**

Problem Set 2

Due: Dec. 17

Note: Please send the solutions to `hsun@mpi-inf.mpg.de` before the deadline.

Problem 1 (5 points) A d-regular graph G is locally invertible if there is a permutation $\psi : [d] \to [d]$, such that the rotation map of G is of the form $\mathrm{Rot}_G(v, i) = (v[i], \psi(i))$, for any $v \in V[G]$ and $i \in [d]$. Show that every d-regular graph is locally invertible.

Problem 2 (6 points) Let G and H be two graphs shown in Figure 1. Calculate (1) $\mathrm{Rot}_{G \circledz H}((1,1),(1,2))$; (2) $\mathrm{Rot}_{G \circledz H}((3,4),(2,2))$; (3) $\mathrm{Rot}_{G \circledz H}((6,2),(3,1))$.

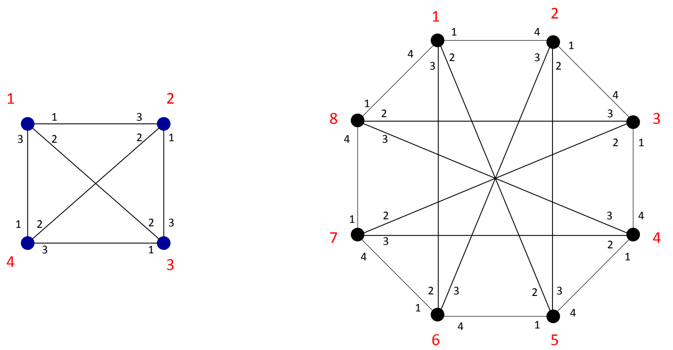

Figure 1: The left graph is H, and the right graph is G.

Problem 3 (4 points) Let $\mathcal{D}$ and $\mathcal{E}$ be two distributions on a set Ω. Show that

$$\Delta(\mathcal{D}, \mathcal{E}) = \frac{1}{2} \cdot \sum_{\alpha \in \Omega} \left| \Pr[\mathcal{D} = \alpha] - \Pr[\mathcal{E} = \alpha] \right| .$$

Problem 4 (5 points) Let $\{X_n\}_{n \geq 1}$ and $\{Y_n\}_{n \geq 1}$ be computationally indistinguishable and $k : \mathbb{N} \to \mathbb{N}$ be a polynomial. Define the probability ensembles $\mathcal{X} = \left\{\overline{X_n}\right\}_{n \geq 1}$ and $\mathcal{Y} = \left\{\overline{Y_n}\right\}_{n \geq 1}$ as follows:

$$\overline{X_n} = X_n^1 \circ \cdots \circ X_n^{k(n)}, \qquad \overline{Y_n} = Y_n^1 \circ \cdots \circ Y_n^{k(n)},$$

where X_n^i's are independent copies of X_n and Y_n^i's are independent copies of Y_n. Show that $\mathcal{X} \sim^{\mathrm{c}} \mathcal{Y}$.