This assignment is due on February 2/4 in your respective tutorial groups. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be written individually without consulting someone else’s solutions or any other source like the web.

**Exercise 1** [Shortest Path:] We covered an external memory algorithms for BFS in undirected graphs in class. BFS is tantamount to shortest paths where all edges have length 1. We now allow edge lengths in \{1, d\} where \(d\) is a small constant. Generalize the algorithm and its analysis.

**Selection:** We are given a \(S\) and an integer \(i\) with \(1 \leq i \leq |S|\). The goal is to find the \(i\)-th largest element in \(S\). \(S\) is ordered by \(\leq\).

**Exercise 2** Recapitulate the sequential solution. We define a recursive procedure \(select(S, i)\).

- randomized solution: if \(|S| = 1\) (and hence \(i = 1\)) return the unique element in \(S\). If \(|S| > 1\), choose a random element in \(S\), call it \(x\), and split \(S\) into \(S_\leq = \{s \in S; s < x\}\), \(\{x\}\), and \(S_\geq = \{s \in S; s > x\}\). If \(i \leq |S_\leq|\), return \(select(S_\leq, i)\), if \(i = |S_\leq| + 1\), return \(x\), and otherwise return \(select(S_\geq, i - |S_\leq| + 1)\). Randomized select works in linear time.

- deterministic solution: divide \(S\) into groups of size 5 and determine the median of each group. Let \(m_1, \ldots, m_k\) be the medians. Let \(m^*\) be the median of the medians; use the procedure recursively to find \(m^*\). Then proceed as in the deterministic solution with \(x = m^*\). Deterministic select works in linear time.

Generalize to external memory, PRAM, and multi-core.