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WS 2010-11

## Models of Computation, an Algorithmic Perspective

Assignment 2

Mon 25.10.2010

This assignment is **due on November 3/5** in your respective tutorial groups. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

**Exercise 1** Let  $G = (V, E)$  be a graph with bounded maximum degree  $\Delta$ , i.e., there is a constant  $C$  such that  $\Delta \leq C$ . Show that there is a synchronous distributed algorithm with time complexity  $O(\log^* |V|)$  that colors  $G$  with  $\Delta + 1$  colors. You may assume that initially all vertices are equipped with distinct labels from  $1 \dots |V|$ . You may perform the following steps.

- a) Modify the algorithm 6-color from the lecture so that it produces a valid coloring for  $G$  by using at most  $2^{3\Delta}$  colors.
- b) Now give an algorithm to recolor the graph that reduces the number of colors used to  $\Delta + 1$  in a constant number of steps.

**Exercise 2** Prove that any synchronous distributed algorithm for coloring any path of  $n$  vertices with 2 colors requires  $\Omega(n)$  rounds.