This assignment is due on December 1/3 in your respective tutorial groups. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be written individually without consulting someone else’s solutions or any other source like the web.

**Exercise 1** An embedding of a graph $G$ in a normed space $(\mathbb{R}, ||.||)$ is called isometric, if it preserves all distances of the vertices in $G$.

a) Show that the complete graph with $n$ vertices $K_n$ can be embedded isometrically in $(\mathbb{R}^d, ||.||)$, where $d = \lceil \log_2 n \rceil$, and an appropriate choice of $||.||$.

b) Show that the cycle $C_{2m}$ with $2m$ vertices can be embedded isometrically in $(\mathbb{R}^d, ||.||)$, where $d = m$, and an appropriate choice of $||.||$.

c) $(\star)$ Show that the above constructions are tight, i.e., there can be no isometric embedding in a normed space $(\mathbb{R}^d, ||.||)$ with smaller dimension.

**Exercise 2** Let $T = (V_T, E_T)$ be a tree with at least 3 vertices. We say that a vertex $v$ in $T$ is central, if we can split $T$ in two trees $T_1 = (V_{T_1}, E_{T_1})$ and $T_2 = (V_{T_2}, E_{T_2})$ such that

- $V_{T_1} \cap V_{T_2} = \{v\}$,
- $E_{T_1} \cup E_{T_2} = E_T$,
- $|V_{T_1}|, |V_{T_2}| \leq \frac{3}{4}|V_T|$.

Show that $T$ has a central vertex.