Universität des Saarlandes

Kurt Mehlhorn, Konstantinos Panagiotou and Reto Spöhel

WS 2010-11

Models of Computation, an Algorithmic Perspective

Assignment 6 Tue 23.11.2010

This assignment is **due on December 1/3** in your respective tutorial groups. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Exercise 1 An embedding of a graph G in a normed space $(\mathbb{R}, \|.\|)$ is called isometric, if it preserves all distances of the vertices in G.

a) Show that the complete graph with n vertices K_n can be embedded isometrically in $(\mathbb{R}^d, \|.\|)$, where $d = \lceil \log_2 n \rceil$, and an appropriate choice of $\|.\|$.

b) Show that the cycle C_{2m} with $2m$ vertices can be embedded isometrically in $(\mathbb{R}^d, \|.\|)$, where $d = m$, and an appropriate choice of $\|.\|$.

c) $(\star)$ Show that the above constructions are tight, i.e., there can be no isometric embedding in a normed space $(\mathbb{R}^d, \|.\|)$ with smaller dimension.

Exercise 2 Let $T = (V_T, E_T)$ be a tree with at least 3 vertices. We say that a vertex v in T is *central*, if we can split T in two trees $T_1 = (V_{T_1}, E_{T_1})$ and $T_2 = (V_{T_2}, E_{T_2})$ such that

- $V_{T_1} \cap V_{T_2} = \{v\}$,
- $E_{T_1} \cup E_{T_2} = E_T$,
- $|V_{T_1}|, |V_{T_2}| \leq \frac{3}{4}|V_T|$.

Show that T has a central vertex.