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Models of Computation, an Algorithmic Perspective

Assignment 9 Tue 14.12.2010

This assignment is **due on January 5/7** in your respective tutorial groups. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Exercise 1 Consider a tree T (on vertex set $V = \{1, ..., n\}$) in the Euler tree representation. For any vertex $r \in V$, we let T_r denote the rooted tree obtained by rooting T at r. For vertices u and v, *least common ancestor* of u and v ($LCA(u, v)$) is defined as the node furthest from the root that is an ancestor of both.

We want to answer the following type of queries: *Given three vertices* $r, u, v \in V$*, what is* $LCA(u, v)$ *in* T_r*?*

a) Design an algorithm that finds the answer to such a query with depth $O(\log n)$ and work $O(n \log n)$. [Hint: The following seemingly unrelated problem might be relevant: *Given an array* $A[1..n]$ *and two indices* i_1 *and* i_2*, what is* $\min_{i_1 \leq j \leq i_2} A[j]$?]

b) Assume now that we want to answer many such queries for the same root r. Design a data structure that represents T_r in such a way that the least common ancestor of two vertices u, v can be found with *constant* depth and work. The construction of the data structure should take only depth $O(\log n)$ and work $O(n \log n)$. [Hint: Precompute the answer for an appropriately chosen set of $O(n \log n)$ queries by dynamic programming!]

Exercise 2 Let $G = (V, E)$ be a planar graph with edge lengths given by $\ell : E \to \mathbb{N}$. Show that for any $0 < \varepsilon < 1/2$, we can preprocess the instance with depth $O(\sqrt{n} \log^2 n + n^{2\varepsilon} \log n)$ and work $O(n^{3(1-\varepsilon)} \log n)$ such that the distance between any two vertices can be computed in sequential time $O(n^{2\varepsilon} \log n)$.