Spectral Graph Theory and Applications **WS 2011/2012**

Problem Set 2

Due: Jan. 11, 2012

Note: Please send the solutions to `hsun@mpi-inf.mpg.de` or `sauerwal@mpi-inf.mpg.de` before the deadline.

Problem 1 Prove that the transition matrix $\mathbf{M}$ of a random walk on an undirected graph G is irreducible if and only if G is connected.

Problem 2 Let $G = (V, E)$ be any undirected graph. Prove that the following three statements are equivalent:

1. The Markov chain corresponding to a random walk with transition matrix $\mathbf{M}$ is **periodic**,
2. -1 is an eigenvalue of $\mathbf{M}$,
3. The graph G is bipartite.

Problem 3 Consider a random walk on a path with nodes $0, 1, \ldots, n$. Prove that for any $0 \leq i < k \leq n$ that

$$\mathbf{H}(i, k) = k^2 - i^2.$$

Hint: First compute $\mathbf{H}(n-1, n)$ with the help of $\mathbf{H}(n, n)$ and then generalize it to $\mathbf{H}(i, k)$ for $i < k \leq n$.

Problem 4 Let $\mathbf{P}$ be a transition matrix which is reversible w.r.t. π. Show that $\mathbf{P}^2$ is also reversible w.r.t. π.