



## Exercises for Algorithmic Game Theory

<http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/AGT/>

### Assignment 2

Deadline: Mo 29.10.2012

When you are asked to provide games with certain properties, the payouts should be **positive integers in**  $[1, 10]$ .

#### Exercise 1 *Two-player zero-sum game*

Consider a two-player zero-sum game in matrix form where no two entries in the matrix are the same.

- Show that there can be at most one PURE Nash equilibrium in such a game.
- Give (small) examples of a game with zero and one Nash equilibria.

#### Exercise 2 *Three-person zero-sum games*

Consider a three-person zero-sum game, which is a game in which the rewards of the three players always sum to zero. Show that finding a Nash equilibrium in such a game is at least as hard as in a general (non-zero-sum) two-player game.

#### Exercise 3 *Quality of Nash equilibria*

- Give a two-player game with two pure Nash equilibria, where both players strongly prefer the same Nash equilibrium
- Give a two-player game with two pure Nash equilibria, where each player strongly prefers a different Nash equilibrium.

#### Exercise 4 *Best response criterion*

- Give an example of a Nash equilibrium in a two-player game where one player uses a pure strategy and one player uses a mixed strategy.
- Determine all Nash equilibria of the following game using the best response criterion. The first player (A) picks a row and the second player (B) picks a column. How many options do you need to check?

$$A = \begin{pmatrix} 3 & 3 \\ 2 & 5 \\ 0 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 4 \\ 2 & 6 \\ 3 & 1 \end{pmatrix}$$

**Hint:** formulate and solve a system of linear equalities for each case to determine the strategies of the players.