



## Exercises for Algorithmic Game Theory

<http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/AGT/>

### Assignment 4

Deadline: Mo 12.11.2012

#### Exercise 1 *VCG mechanisms*

Show formally that a VCG mechanism is incentive compatible, that is, no player can benefit by changing its (claimed) valuation function.

#### Exercise 2 *Clarke's pivot rule*

Show that a second-price auction is a VCG-mechanism with Clarke's pivot rule.

**Remark:** We showed already in class that it is a VCG-mechanism.

#### Exercise 3 *Reverse auction*

Assume that we want to **buy** an item from a seller with lowest cost. The valuations of the sellers are as follows:  $v_i(i \text{ wins}) = w_i < 0$  and  $v_i(j \text{ wins}) = 0 \forall j \neq i$ .

- Give a VCG mechanism for this auction. Who wins? What are the payments?
- Show that the mechanism is equivalent to adding  $\max_i(-w_i)$  to every bid, running the standard Vickrey auction, and adjusting the prices by subtracting  $\max_i(-w_i)$  again.

#### Exercise 4 *Bilateral Trade*

In this problem a seller holds an item and values it at some value  $w_s$  and a buyer wants the item and values it at some value  $w_b$ . Let  $A = \{ \text{"trade"}; \text{"no-trade"} \}$  and define

$$v_s(\text{trade}) = -w_s \quad (1)$$

$$v_b(\text{trade}) = w_b \quad (2)$$

$$v_s(\text{no-trade}) = v_b(\text{no-trade}) = 0 : \quad (3)$$

Let  $(f; p_s; p_b)$  be a VCG-mechanism.

- What are the conditions for outcome "trade"?
- Define the functions  $h_s$  and  $h_b$  such that  $p_b = p_s = 0$  in the case of no-trade. What are  $p_b$  and  $p_s$  in the case of trade?