Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Algorithmic Game Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/AGT/

Assignment 7 Deadline: Mo 3.12.2012

Exercise 1 *Return of the double-minded bidders*

Let us consider a seemingly easier case of double-minded bidders. Consider a setting where each player i wants either a specific set S_i^* at value v_i^* or **all** the items at value $\tilde{v}_i$. Modify the greedy mechanism for single-minded bidders as follows.
First, hold a Vickrey auction for the whole set of items. Compare this to the output of the greedy mechanism with subsets S_i^*. Take the output which gives the highest social welfare and use the payments of the corresponding mechanism.
Show that this mechanism is not truthful.

Exercise 2 *The greedy algorithm for makespan scheduling*

We have two machines and three jobs. The job sizes are $2, 1+\varepsilon, 1+\varepsilon$. The greedy algorithm (List Scheduling) considers the jobs one by one, in order of decreasing size, and assigns each job to the machine that currently has the lowest load, preferring a faster machine in case of a tie.

- Give the output of the greedy algorithm if the speeds are $1+\varepsilon, 1$.
- Give the output of the greedy algorithm if the speeds are $1-\varepsilon, 1$.
- Can we use this algorithm as part of a truthful mechanism? That is, can you give a payment function so that the resulting mechanism is truthful?

Exercise 3 *Calculation of the payment function*

Truthful Mechanism 1 from class:

- Consider all optimal allocations based on the bids $(b_1, \ldots, b_m)$
- Use the allocation in which $(q_1, \ldots, q_m)$ is lexicographically minimal

Questions:

a) Starting from some speed, bidder i will receive all jobs. Give an upper bound for it.
b) Assume that we have run Truthful Mechanism 1 for a particular input. Give an efficient way to find a range of bids for player i for which the resulting allocation is optimal.
c) Give an algorithm to calculate the payment function for bidder i.
d) What is the running time of your algorithm? You may use that there exist m^n different allocations of n jobs to m machines.