Universität des Saarlandes
FR 6.2 Informatik

Sayan Bhattacharya, Rob van Stee **SS 2011**

Exercises for Algorithmic Game Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/AGT/

Assignment 8 Deadline: Mo 10.12.2012

Exercise 1 *Scheduling to minimize the total weighted completion time*

Consider the example discussed in class:

Job	Weight	Size
1	1	1
2	w	p

Let $pw < 1$. The machines have speeds $1, s$. Assume that the truthful mechanism TM does not use the optimal assignment for $s = w + 1$. Give a lower bound for its approximation ratio.

Exercise 2 *Price of stability and price of anarchy*

Design a game for which the price of anarchy is 10 and the price of stability is 1. The game should have exactly two players and two strategies for every player, and be symmetric (the payout matrix of player 1 is the transposed payout matrix of player 2).

Exercise 3 *Machine covering*

Consider machine scheduling where the objective is to *maximize* the minimum load. **Jobs** are selfish (i.e. controlled by selfish agents) and want to be on a machine with low load. The load of a machine is the total size of the jobs that are on this machine divided by its speed.

a) Show that the price of anarchy is unbounded already on two machines if one machine is at least twice as fast as the other.
b) The price of stability is the ratio of the cost of the best Nash equilibrium to the optimal cost. When is the price of stability unbounded for two machines?

Exercise 4 *Machine covering 2*

Consider the same scheduling problem as in the previous exercise, this time on m identical machines and with m jobs of size 1. Consider the following fully mixed Nash equilibrium: for every player i and machine j, the probability that player i goes to machine j is $1/m$.

a) What is the number of job assignments with nonzero cover (minimum load)?
b) What is the total number of possible assignments?
c) What is the probability that the cover is nonzero? What is the expected cover?
d) Give a lower bound for the mixed price of anarchy (value of the optimal cover divided by the cover of the worst mixed NE).