Universität
des
Saarlandes
FR 6.2 Informatik

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Exercises for Algorithmic Game Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/AGT/

Assignment 9 Deadline: Mo 17.12.2012

Exercise 1 *Pigou's example*

Pigous example is the selfish routing game discussed in class, with two parallel links. Modify Pigous example so that the lower edge has cost function $c(x) = x^d$ for some $d > 1$ (the top edge still has cost function $c(x) = 1$). What is the Price of Anarchy of the resulting selfish routing network, as a function of d?

Exercise 2 *Undirected Shapley network game*

Consider the following instance of a network creation game. There are n players with n different source verticess$s_1, s_2, \dots, s_n$ and a common target vertex t. Every source vertex s_i is connected to the target vertex t by an edge with cost $1/i$. Moreover, every source vertex s_i is connected to a vertex v by an edge with cost 0, and this vertex v is connected to the target vertex t by an edge with cost $1 + \varepsilon$. As opposed to the instance discussed in the lecture, we now assume that all edges are undirected. What is the Price of Stability?

Exercise 3 *Load balancing on two identical machines*

The **load balancing game** is defined as follows. There are m parallel (identical) machines and n tasks. The tasks are selfish and each task selects a machine to run on. Their objective is to be run on a machine with the lowest possible load. Let G be any instance of this game with $n = 3$ tasks and $m = 2$ identical machines. Show that any pure Nash equilibrium for G is optimal, i.e. $\textsc{cost}(A) = \textsc{opt}(G)$ for any equilibrium assignment A.

Exercise 4 *Load balancing on identical machines*

The same game as in the previous exercise. Show, for every $m \in \mathbf{N}$, there exists an instance G of the load balancing game with m identical machines and $2m$ tasks that has a Nash equilibrium assignment $A : [n] \to [m]$ with

$$cost(A) = \left(2 - \frac{2}{m+1}\right) \cdot \textsc{opt}(G)$$