Theorem 1 *For $\alpha < 1$ and $\alpha > 2$, PoS=1. Else, PoS is at most $4/3$.*

Proof First part follows from previous lemmas: NE = optimal solution.

Let $1 < \alpha < 2$. Consider ratio of costs. We get

$$\begin{aligned}
\frac{NE}{\text{OPT}} &= \frac{star}{K_n} && (1)\\
&= \frac{2n(n-1) + (\alpha-2)(n-1)}{2n(n-1) + (\alpha-2)n(n-1)/2} && (2)\\
&= \frac{2n(n-1) - 2(n-1) + \alpha(n-1)}{2n(n-1) - n(n-1) + \alpha n(n-1)/2} && (3)\\
&= \frac{(2n-2)(n-1) + \alpha(n-1)}{n(n-1) + \alpha n(n-1)/2} && (4)\\
&= \frac{2n-2+\alpha}{n+\alpha n/2} \qquad \text{is maximized for } \alpha \to 1 && (5)\\
&< \frac{2n-1}{3n/2} \qquad = \frac{4n-2}{3n} < \frac{4}{3}\ . && (6)
\end{aligned}$$

The first inequality follows because $\alpha > 1$. $\square$