Basic Mathematical Techniques for Computer Scientists

Propositional Logic, Part Two

October 29, 2012

Recap

- What is a *proof*?

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  - *Axioms*

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  - *Logical deductions*

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  - From axioms to a proposition, using logical deductions.

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Questions?

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  - Why is this a good thing?

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Example: Number theory

- A candidate axiom for number theory
  - (The study of integers, primes, and so on.)

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  - What it means to divide one integer by another
  - What a prime number is

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- A candidate axiom for number theory
  - (The study of integers, primes, and so on.)
- Assume we have defined
  - What it means to divide one integer by another
  - What a prime number is
- "If a prime number p divides the product ab of two integers a and b, then p divides at least one of $\{a, b\}$."
  - In symbols: $\forall$prime $p\ \ \forall a, b \in \mathbb{I}\ \ p \mid ab \implies (p \mid a) \vee (p \mid b)$

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  - Depends on what "reasonable" means to you . . .
  - This does seem to be a "fundamental" property of integers

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  - This does seem to be a "fundamental" property of integers
    - I mean, how much more basic can we get than this?

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  5. ... etc.

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  - *Derive* it as a **theorem** from more basic axioms.
- So the above proposition is indeed true
- And its truth depends on very basic axioms
- For some commonly accepted but vague notion of "basic"

Axioms

Example: Euclid's axioms for plane geometry (Elements, Book I)

- Five *postulates*
- Five *common notions*

Axioms

Example: Euclid's axioms for plane geometry (Elements, Book I)

- Five *postulates*
  1. One can draw a unique straight line through any given pair of points.
  2. One can extend any (finite) line segment to a unique (infinite) straight line.
  3. Given any point c and any length r, one can draw a unique circle which has centre c and radius r.
  4. All right angles are equal to one another.
  5. For any line l and a point p not on l, there is exactly one line through p which is parallel to l.

Axioms

Example: Euclid's axioms for plane geometry (Elements, Book I)

- Five *common notions*
  1. Things that are equal to the same thing are also equal to one another.
  2. If equals are added to equals, then the wholes are equal.
  3. If equals are subtracted from equals, then the remainders are equal.
  4. Things that coincide with one another equal one another.
  5. The whole is greater than the part.

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- Reasonable assumptions to make about geometric objects on an ideal "flat" plane

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- *Not* necessarily true for
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  - Other mathematical systems

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- Reasonable assumptions to make about geometric objects on an ideal "flat" plane
- *Not* necessarily true for
  - Other geometries which are equally (perhaps more) "real"
    - The fifth (parallel) postulate does not hold for geometry on spherical surfaces.
  - Other mathematical systems
    - The fifth common notion does not hold for infinite sets.
    - If we take "greater" to mean "contains more elements".

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  - We assume commonly known stuff
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  - E.g: In a first course on Topology or Group Theory
  - You will argue many things starting from the respective axioms
  - This is to get practice thinking in the new way

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- Exception: When learning a new sub-field of mathematics
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  - Rarely done in a second course or later

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  - In this course, and usually in others
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  - Don't assume the solution itself!
  - ... or something very close to it
    - That is cheating!!
- When in doubt, explicitly declare your assumptions
- Check if the question mentions "axioms" or some such

Logical deductions

- Ways of combining axioms and true propositions
  - To form new true propositions

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Logical deductions

- Ways of combining axioms and true propositions
  - To form new true propositions
- Also called *Rules of Inference*
- There are many such rules
  - Some of these have fancy Latin names
  - Most of them are just “common sense”

Some special kinds of propositions

Tautology

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- Examples?
- The truth of a tautology comes
  - From the principles of propositional logic
  - Not from any “outside” information

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- The falsity of a tautology comes
  - From the principles of propositional logic
  - Not from any “outside” information

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Contingent proposition

- A proposition which is neither a tautology nor a contradiction

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Some special kinds of propositions

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- Examples?
- Not very special, really . . .

Rules of replacement

- A rule used to transform some part of a logical expression
  - Replace some part by an *equivalent* part

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  - Replace some part by an *equivalent* part
- Many rules have names
  - We will see a few
  - Many of these are “common sense”

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Double negation

- Two rules

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- Can replace
  - A *anywhere* with $(\neg\neg A)$
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  - $(\neg\neg A)$ *anywhere* with A
- Why are these OK?

Rules of replacement

Commutativity

- Four rules, two each for AND and OR

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- Four rules, two each for AND and OR
- Can replace
  - $A \wedge B$ *anywhere* with $B \wedge A$
    - And conversely
  - $A \vee B$ *anywhere* with $B \vee A$
    - And conversely

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De Morgan's laws

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  - $\neg(A \wedge B)$ *anywhere* with $\neg A \vee \neg B$
    - And conversely
  - $\neg(A \vee B)$ *anywhere* with $\neg A \wedge \neg B$
    - And conversely

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De Morgan's laws

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- Can replace
  - $\neg(A \wedge B)$ *anywhere* with $\neg A \vee \neg B$
    - And conversely
  - $\neg(A \vee B)$ *anywhere* with $\neg A \wedge \neg B$
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- Used to *infer* new propositions from old

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Disjunctive Syllogism

$$\frac{P \vee Q, \quad \neg P}{Q}$$

Rule of Inference

Disjunctive Syllogism

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- "If everything above the bar is true, then the thing below the bar is also true."
  - Above the bar are the *premises*, below the bar the *conclusion*

Rule of Inference

Disjunctive Syllogism

$$\frac{P \vee Q, \quad \neg P}{Q}$$

- If at least one of $\{P, Q\}$ is true, and P is *not* true, then Q is true.

Rule of Inference

Disjunctive Syllogism

$$\frac{P \vee Q, \quad \neg P}{Q}$$

- If at least one of $\{P, Q\}$ is true, and P is *not* true, then Q is true.
- Example:
  - I drank coffee today, or I drank tea today.
    $(P \vee Q)$
  - I did not drink coffee today. $(\neg P)$
  - So: I drank tea today. (Q)

Rule of Inference

Disjunctive Syllogism

$$\frac{P \vee Q, \quad \neg P}{Q}$$

- Seems legit?

Rule of Inference

Disjunctive Syllogism

$$\frac{P \vee Q, \quad \neg P}{Q}$$

- Seems legit?
- What would be a *proof* for this?

Rule of Inference

Disjunctive Syllogism

$$\frac{P \vee Q, \quad \neg P}{Q}$$

- Seems legit?
- What would be a *proof* for this?
- One way: the truth table of $P \vee Q$

Rule of Inference

Modus Ponens

$$\frac{P, \quad P \implies Q}{Q}$$

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- If P is true and $(P \implies Q)$ is true, then Q is true.

Rule of Inference

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- Example:
  - If it is Monday and it is not a holiday, then we have class. $(P \implies Q)$

Rule of Inference

Modus Ponens

$$\frac{P, \quad P \implies Q}{Q}$$

- If P is true and $(P \implies Q)$ is true, then Q is true.
- Example:
  - If it is Monday and it is not a holiday, then we have class. $(P \implies Q)$
  - It is Monday, and it is not a holiday. (P)
  - So: We have class. (Q)

Rule of Inference

Modus Ponens

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Modus Ponens

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- Another way: A "formal" proof

Rule of Inference

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  - End up with the conclusion

Rule of Inference

Modus Ponens

$$\frac{P, \quad P \implies Q}{Q}$$

- Another way: A "formal" proof
  - Start with the premises
  - Use the various rules we know
  - End up with the conclusion
  - Done!

Rule of Inference

Modus Ponens

$$\frac{P, \quad P \implies Q}{Q}$$

Table: A formal proof of Modus Ponens[a]

	Proposition	Derivation
1	$P \implies Q$	
2	P	
3	$\neg P \vee Q$	
4	$\neg\neg P$	
5	Q	

[a] Stolen from the Wikipedia entry on this rule.

Rule of Inference

Modus Ponens

$$\frac{P, \quad P \implies Q}{Q}$$

Table: A formal proof of Modus Ponens[a]

	Proposition	Derivation
1	$P \implies Q$	Given
2	P	
3	$\neg P \vee Q$	
4	$\neg\neg P$	
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Modus Ponens

$$\frac{P, \quad P \implies Q}{Q}$$

Table: A formal proof of Modus Ponens[a]

	Proposition	Derivation
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Rule of Inference

Modus Ponens

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Table: A formal proof of Modus Ponens[a]

	Proposition	Derivation
1	$P \implies Q$	Given
2	P	Given
3	$\neg P \vee Q$	Homework 1
4	$\neg\neg P$	
5	Q	

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Rule of Inference

Modus Ponens

$$\frac{P, \quad P \implies Q}{Q}$$

Table: A formal proof of Modus Ponens[a]

	Proposition	Derivation
1	$P \implies Q$	Given
2	P	Given
3	$\neg P \vee Q$	Homework 1
4	$\neg\neg P$	Double Negation
5	Q	

[a] Stolen from the Wikipedia entry on this rule.

Rule of Inference

Modus Ponens

$$\frac{P, \quad P \implies Q}{Q}$$

Table: A formal proof of Modus Ponens[a]

	Proposition	Derivation
1	$P \implies Q$	Given
2	P	Given
3	$\neg P \vee Q$	Homework 1
4	$\neg\neg P$	Double Negation
5	Q	Disjunctive Syllogism

[a] Stolen from the Wikipedia entry on this rule.

Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

- If $P \implies Q$ is true, and Q is *false*, then P is false.

Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

- If $P \implies Q$ is true, and Q is *false*, then P is false.
- Example:
  - If I am ill then I don't come to class.
    $(P \implies Q)$
  - I come to class. $(\neg Q)$
  - So: I am not ill. $(\neg P)$

Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

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Rule of Inference

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Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

- Seems legit?
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- One way: the truth table of $P \implies Q$

Rule of Inference

Modus Tollens

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Table: A formal proof of Modus Tollens[a]

	Proposition	Derivation
1	$P \implies Q$	
2	$\neg Q$	
3	$\neg P \vee Q$	
4	$\neg P$	

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Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

Table: A formal proof of Modus Tollens[a]

	Proposition	Derivation
1	$P \implies Q$	Given
2	$\neg Q$	
3	$\neg P \vee Q$	
4	$\neg P$	

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Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

Table: A formal proof of Modus Tollens[a]

	Proposition	Derivation
1	$P \implies Q$	Given
2	$\neg Q$	Given
3	$\neg P \lor Q$	
4	$\neg P$	

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Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

Table: A formal proof of Modus Tollens[a]

	Proposition	Derivation
1	$P \implies Q$	Given
2	$\neg Q$	Given
3	$\neg P \vee Q$	Homework 1
4	$\neg P$	

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Rule of Inference

Modus Tollens

$$\frac{P \implies Q, \quad \neg Q}{\neg P}$$

Table: A formal proof of Modus Tollens[a]

	Proposition	Derivation
1	$P \implies Q$	Given
2	$\neg Q$	Given
3	$\neg P \vee Q$	Homework 1
4	$\neg P$	Disjunctive Syllogism

[a] Stolen from the Wikipedia entry on this rule.

Thank You!