max planck institut informatik

SAARLAND UNIVERSITY

COMPUTER SCIENCE

Prof. Dr. Benjamin Doerr **Winter 2012/13**

Exercises for Randomized Methods in Computer Science

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws12/rmcs/

Assignment 12 Due: Wednesday, February 6, 2013

Exercise 1 Read the first ten pages from Joel Spencer's "Nine Lectures on Random Graphs". Pay close attention to the second moment method used in Theorems 4.1 and 4.2 (this includes the preliminary arguments at the end of the preceding section). Feel free to skip Theorems 4.4. and 4.5. Take cursory note of Sections 1 and 2 of Lecture 2, but study intensively Section 3 (connectivity threshold).

Exercise 2 Complete the proof of the theorem on the connectivity threshold by proving the missing statement that the probability for having a connected component of at least two and at most $n/2$ vertices, is $o(1)$.

Exercise 3 Graphs with high girth and high chromatic number. This is a classic of random graph theory, so you have to do it. A famous problem in graph theory is whether there are graphs that have arbitrary high coloring number (minimum number of colors to color the vertices such that adjacent vertices do not get the same color) and arbitrary high girth (length of a shortest cycle). In simple words, can a graph with no short cycles (locally looking like a tree) still need many colors to be colored? The answer is yes, but you will have a hard time constructing an example by hand (try it, if you succeed, you may skip this exercise).

Let $\ell \in \mathbb{N}$ and $\theta < 1/\ell$. Let G be an random graph on n vertices with edge probability $p = n^{\theta-1}$.

a) Compute the expected number of circles of length ℓ or less in G (an upper bound suffices). Conclude that with high probability, there are less than $n/2$ such circles.

b) Let $x = \lceil (3/p)\ln(n) \rceil$. Compute that with high probability, there is no independent set of x or more vertices in G.

c) G is not yet what we want, but almost. Obtain G^* from G by removing one vertex from every cycle of length ℓ or less. Argue that this graph has girth greater than ℓ and coloring number at least $n^\theta/(6\ln n)$. For the latter remember the elementary connection between coloring number and independent sets.

Hence by taking ℓ and n large enough, we obtain graphs with arbitrary girth and coloring number.