max planck institut informatik

SAARLAND UNIVERSITY
COMPUTER SCIENCE

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Exercises for Randomized Methods in Computer Science

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws12/rmcs/

Assignment 7 Due: Wednesday, December 12, 2012

Exercise 0 Read cursory the second chapter of the book A.E. Eiben and J.E. Smith, Introduction to Evolutionary Computing, Springer, Natural Computing Series 1st edition, 2003, ISBN: 3-540-40184-9 Corr. 2nd printing, 2007, ISBN: 978-3-540-40184-1. It is freely available at `http://www.cs.vu.nl/ gusz/ecbook/Eiben-Smith-Intro2EC-Ch2.pdf`. You shouldn't learn everything by heart, but get a feeling for the language and the general thinking in this area.

Exercise 1 Fitness-based partition method: A simple, but sometimes useful tool is the following.

Theorem 1 *Consider a simple randomized search heuristic with population size one that aims at maximizing a given function $f : S \to \mathbb{R}$. Let this randomized search heuristic be elitist, that is, it does not accept worse solutions. Let $A_1 \dot\cup \ldots \dot\cup A_m$ be a partition of the search space S into disjoint sets such that for all $i, j \in [m]$, $i < j$, we have that for all $x \in A_i$ and $y \in A_j$ we have $f(x) < f(y)$. Assume further that A_m only contains individuals with maximal fitness. Assume that the randomized search heuristic is such that if the current population is an individual from A_i, $i < m$, then the probability that in one iteration an individual in some A_j, $j > i$, is generated, is at least p_i.*

Then the expected number of iterations performed until an optimal individuum is found, is at most $\sum_{i=1}^{m-1} p_i^{-1}$.

a) Prove the above theorem.

b) Use the above theorem to prove that the (1+1) EA finds the optimum of the leadingones function $f : \{0,1\}^n \to [0..n]; x \mapsto \max\{i \in [0..n] \mid \forall j \in [i] : x_j = 1\}$ in $O(n^2)$ iterations.

c) tricky Use the above theorem to show that the (1+1) EA finds the optimum of any linear function $f : \{0,1\}^n \to \mathbb{R}; x \mapsto \sum_{i=1}^{n} a_i x_i$ in $O(n^2)$ iterations. Hint: Use the suitable fitness-based partition of the search space.