max planck institut informatik

SAARLAND UNIVERSITY

COMPUTER SCIENCE

Prof. Dr. Benjamin Doerr **Winter 2012/13**

Exercises for Randomized Methods in Computer Science

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws12/rmcs/

Assignment 9 Due: Wednesday, January 2, 2013

Exercise 1 (*easy*) Use Wald's equation to give bounds for "how often to you have to roll a standard die until the sum of the rolled numbers is at least 100".

Exercise 2 (*easy with all the hints*) In this exercise, we prove the drift theorem given in the lecture.

a) To this aim, we need an elementary trick, that often comes useful. Let X be a random variable taking non-negative integer values. Assume that $E(X)$ is finite. Then $E(X) = \sum_{i=1}^{\infty} \Pr(X \geq i)$.

b) Let $X_0, X_1, X_2, \ldots$ be random variables taking values in some finite subset $S \subset \{0\} \cup [1, \infty)$. Assume that X_0 takes some positive value x_0 with probability 1. Assume that there is a $\delta > 0$ such that for all $t \in \mathbb{N}$, $s \in S$, we have $E(X_t | X_{t-1} = s) \leq (1-\delta)s$. Give a clean proof for the fact that $E(X_t) \leq (1-\delta)^t x_0$ for all $t \in \mathbb{N}_0$.

c) Continuing the notation from (b), let $T := \min\{t \in \mathbb{N}_0 \mid X_t = 0\}$. Prove that $E(T) \leq (1/\delta)(1 + \ln(x_0))$.

d) Still continuing from above, show that the probability that T exceeds $(1/\delta)(c + \ln(x_0))$ is at most $\exp(-c)$.

Note: If you have difficulties with some estimates and only get slightly inferior bounds, don't worry.

Exercise 3 (*easy*) Use the drift theorem proven above to analyze (once again) the coupon collector (expected time, tail bounds).

Exercise 4 (*easy*) Use the drift theorem to show that the (1+1) EA finds an optimum of any linear function in time $O(n \log n)$, when the coefficients are polynomially bounded.

Exercise 5 (*easy*) Imagine you arrange a chain of n dominos so that, once you're done, you can have them all fall sequentially in a nice manner by knocking down the first domino. When setting this up, each time you place a domino, there is a small chance p that it falls, knocking down all other dominoes already placed. In this case, you have to start again. How often, in expectation, you have to place a domino until you completed the n domino arrangement? [from the Mitzenmacher/Upfal book]