Topics in Approximation Algorithms: Assignment 1

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Problem 1. Give an instance of the set cover problem where the greedy algorithm returns a solution with cost at least $\Omega(\log n)$ times the cost of the optimal solution, n being the number of elements in the universe.

Problem 2. (Williamson-Shmoys Book. Exercise 1.4(b))

In the *uncapacitated facility location* problem, we have a set of clients D and a set of facilities F. For each client $j \in D$ and facility $i \in F$, there is a cost c_{ij} of assigning client j to facility i. Furthermore, there is a cost f_i associated with each facility $i \in F$. The goal is to choose a subset of facilities $F' \subseteq F$ so as to minimize the total cost of the facilities in F' and the cost of assigning each client $j \in D$ to the nearest facility in F'. In other words, we wish to find F' so as to minimize $\sum_{i \in F'} f_i + \sum_{j \in D} \min_{i \in F'} c_{ij}$.

Give an $O(\log |D|)$ approximation algorithm for the uncapacitated facility location problem.

Problem 3. (Williamson-Shmoys Book. Exercise 1.6(b))

In the *node-weighted Steiner tree* problem, we are given as input an undirected graph $G = (V, E)$, node weightes $w_i \geq 0$ for all $i \in V$, edge costs $c_e \geq 0$ for all $e \in E$, and a set of terminals $T \subseteq V$. The cost of a tree is the sum of the weights of the nodes plus the sum of the costs of the edges in the tree. The goal is to find a minimum-cost tree that spans all the terminals in T.

Give a greedy $O(\log |T|)$ approximation algorithm for the node-weighted Steiner tree problem.

Problem 4. In class, we showed that the greedy algorithm gives a H_n approximation algorithm for set cover. Now consider a simpler setting of *unweighted set cover* where each set has unit weight (i.e. $w_j = 1$), so the greedy algorithm simply tries to cover as many elements as possible in each step. Define parameter $s = \max_j |S_j|$. Show that the greedy algorithm gives H_s approximation for unweighted set cover. This is an improvement when $s << n$.