

Eric Berberich, Michael Kerber

WS 2013

Excercises Computational Geometry

<http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/>

Sheet 1

Deadline: 22.10.2013, 10:00am

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam.
40 points correspond to 100%; you can get up to 10 bonus points.

Exercise 1 (10 pts)

- Give an example of a non-convex polyhedron with Euler number 2.
- Give an example of a polyhedron with Euler number -2 . Describe a method to construct a polyhedron with Euler number $2 - 2g$ for any integer $g \geq 0$.

Exercise 2 (10 pts)

The integer lattice is given by the points $\mathbb{Z} \times \mathbb{Z}$. The *Body-centered cubic* (BCC) lattice is the union of the integer lattice and a copy of itself shifted by $(\frac{1}{2}, \frac{1}{2})$:

$$(\mathbb{Z} \times \mathbb{Z}) \cup ((\frac{1}{2} + \mathbb{Z}) \times (\frac{1}{2} + \mathbb{Z}))$$

(informally, take a integer lattice and place an additional point at the center of each cube). Compute the packing densities of the integer lattice and the BCC lattice.

Exercise 3 (10 pts)

- Show that the convex hull is a convex set.
- For a point set in the plane with at least four points, show that S can be partitioned in to (disjoint) sets A and B such that the convex hull of A and the convex hull of B intersect.
- For a point set in $\mathbb{R}^3$ with at least five points, show that S can be partitioned in to (disjoint) sets A and B such that the convex hull of A and the convex hull of B intersect.

Exercise 4 (10 pts)

Change the incremental algorithm such that the time for finding the tangent lines leads to a complexity $O(n)$ when summing over all points. What is the complexity of computing the convex hull using this improved search method?

Exercise 5 (10 pts)

For a set S of points in the plane, let ℓ be the line that minimizes the maximal distance to S . Show that this line can be computed in $O(n^2)$ time. (Hint: The convex hull of S will be useful).