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Excercises
Computational Geometry

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/

Sheet 2 Deadline: 29.10.2013, 10:00am

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam. 40 points correspond to 100%; you can get up to 15 bonus points.

Exercise 1 (*10+5 pts*)
Consider a $(k+1)$-tuple of polygons $(E, H_1, \ldots, H_k)$ such that each H_i lies in the interior of E (without touching the boundary) and the H_i are pairwise disjoint. The set

$$P := E \setminus \bigcup_{i=1}^{k} H_i$$

is called a *polygon with k holes*.
Note that P decomposes the space into $k+2$ disjoint regions (why?). The interior of P is the interior of E that does not lie in any hole. A triangulation of P is a triangulation of its interior. The number of vertices of P is the sum of the vertices of $E, H_1, \ldots, H_k$.

a) Show that polygons with 1 hole can always be triangulated.

b) (Bonus): Extend your proof to arbitrary k.

c) Draw examples of polygons with 1,2, and 3 holes. For each, give 3 different triangulations. Count the number of triangles.

d) Formulate a conjecture about the number of triangles in a triangulation of a polygon with k holes having n vertices in total.

Exercise 2 (*10 pts*)
We consider tetrahedralizations of the unit cube spanned by the vertices $\{0,1\}^3$. For notational convenience, we define $\mathbf{0} := (0,0,0)$, $\mathbf{1} := (1,1,1)$, $e_1 := (1,0,0)$ and e_2 and e_3 likewise.
Define a *corner cut at* $\mathbf{0}$ as follows: Insert the diagonals e_1e_2, e_1e_3 and e_2e_3 and remove the tetrahedron spanned by $\{\mathbf{0}, e_1, e_2, e_3\}$ from the cube. A corner cut at any other vertex v can be defined the same way: connect the three neighbors of v by diagonals and remove the tetrahedron defined by the four points.

a) Recall the tetrahedralization of the cube from the lecture. Argue that the six tetrahedra are congruent.

b) Consider the unit cube after cutting the corners **0** and **1**. Argue that no further corner cut is possible. Tetrahedralize the remaining part with four tetrahedra, yielding six in total (including the corners). Show that the six tetrahedra are not congruent.

c) Give a tetrahedralization of the cube into 5 tetrahedra.

d) For a tetrahedron spanned by $\{\mathbf{0}, a, b, c\}$ with $a, b, c \in \mathbb{R}^3$, its volume is given by

$$V = \left| \frac{a(b \times c)}{6} \right| .$$

Prove that there cannot exist a tetrahedralization of the unit cube into 7 tetrahedra.

Exercise 3 (*10 pts*) Describe an $O(n^2)$-algorithm to triangulate a polygon.

Exercise 4 (*10 pts*)

a) For any n, give an example of a polygon with n vertices which permits exactly one triangulation.

b) For any n, give an example of a polygon with n vertices which permits exactly two triangulations.

Exercise 5 (*10 pts*)

a) Prove the *Fortress Theorem*: For covering the **exterior** of a polygon P with n vertices, it suffices to place $\lceil n/2 \rceil$ guards on the boundary of the polygon, and there are examples where $\lceil n/2 \rceil$ such guards are necessary.

b) Consider the case that guards can be placed anywhere in the exterior. Show an example where $\lceil n/3 \rceil$ guards are necessary to cover the exterior area.