Eric Berberich, Michael Kerber **WS 2013**

Excercises
Computational Geometry

http://www.mpi-inf.mpg.de/departments/d1/teaching/ws13/ComputationalGeometry/

Sheet 3 Deadline: 05.11.2013, 10:00am

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam. 40 points correspond to 100%; you can get up to 15 bonus points.

Exercise 1 (*10 pts*)
A triangulation of a point set S is a partition of the plane determined by a maximal set of noncrossing edges whose vertex set is S.

a) Show that the union of these triangles is the convex hull of S.

b) Design a sweep line algorithm for computing a triangulation of a point set S.

Exercise 2 (*10+5 pts*) Discuss in detail the basic insertion function of the DCEL. That is, given a subdivision S as DCEL and a new curve c to be inserted into the DCEL. Precondition: The interior of c lies completely in a face of the DCEL. Which update steps are needed to make the DCEL a valid one of $S \cup \{c\}$? Hint: You might first think about the case which steps are needed to insert a single point. Bonus: Discuss the case of removing an edge from a DCEL.

Exercise 3 (*10 pts*)

a) Give examples of DCELs where for some edge e, the faces IncidentFace(e) and IncidentFace(Twin(e)) are the same.

b) Give an example of a non-empty DCEL of a subdivision where Twin(e) = Next(e) holds for every halfedge of e. What is the maximal number of faces that the subdivision can have?

Exercise 4 (*10 pts*) Let S be a subdivision of complexity n and P be a set of m points. Give a plane sweep algorithm that computes for every point in which face of S it is contained. Show that your algorithm runs in $O((n+m)\log(n+m))$ time.

Exercise 5 (*10 pts*)
We are given a set of cars $\mathcal{C} = \{C_1, \ldots, C_n\}$ that move in a plane desert with constant speed: $C_k = \langle p_k, \vec{v}_k \rangle$, where p_k is the initial position of a car at time $t = 0$ and $\vec{v}_k$ is its velocity vector. Determine, for each time $t \geq 0$, the nearest car to the ranger's stations located at $b = (x_0, y_0)$.